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Short Note

Orientation-Independent, Nongeometric-Mean Measures of Seismic Intensity from Two Horizontal Components of Motion

by David M. Boore

Abstract New measures of spectral intensity based on the horizontal components of ground shaking are introduced. These new measures are independent of the in situ orientation of the recordings and encompass the full range of spectral amplitudes over all possible rotation angles. Unlike previously introduced measures that are also orientation independent, no geometric means are used in the computation of the new measures. The new measures based on fiftieth percentile values of the response spectra show small but systematic increases (to a factor of about 1.07 at a 10 sec period) compared to the comparable geometric-mean measure.

Introduction

Boore *et al.* (2006) introduced measures of horizontal ground motion that are not dependent on the in situ orientations of the sensors. These measures are based on the geometric-mean response spectra of two horizontal components of ground motion rotated in small increments over a 90° range. One measure uses a period-dependent rotation angle; this measure is denoted GMRotDnn, where D stands for period-dependent rotation angle and nn is the fractile of the geometric means for all rotation angles sorted by amplitude (e.g., GMRotD50 is the median value and GMRotD100 is the largest geometric mean over all rotation angles). An unappealing feature of GMRotDnn is that the rotation angle is usually a function of oscillator period. Boore *et al.* (2006) introduced a measure of response spectra that was also independent of the in situ orientations of the sensors but corresponded to a single rotation angle for all oscillator periods. This measure was denoted GMRotInn. GMRotI50 was adopted for use in the ground-motion prediction equations (GMPEs) developed as part of the Pacific Earthquake Engineering Research (PEER) center's Next Generation Attenuation of Ground Motions (NGA) project (Chiou *et al.*, 2008). In addition to a median measure of ground-motion intensity, users (including suggested building codes, such as that of the Building Seismic Safety Council, 2009) have expressed a desire for the maximum spectral amplitude over all rotation angles; Beyer and Bommer (2006), Campbell and Bozorgnia (2007), Watson-Lamprey and Boore (2007), and Huang *et al.* (2008) provided equations to convert GMRotI50 to maximum spectral amplitude. Note that the maximum spectral amplitude does not use geometric means, whereas the GMRotI50 median measure of seismic intensity does use geometric means. The purpose of this note is to introduce and investigate measures of horizontal-component seismic intensity that represent any fractile in a consistent way without computing geometric means and yet still be independent of the in situ orientations of the recorded ground motions. Following the notation used for GMRot, these new measures are denoted RotInn and RotDnn. RotD50 has values similar to GMRotI50, and RotD100 is identical to the spectral maximum over all rotation angles. The ongoing NGA-East project to develop GMPEs in eastern North America is considering the use of RotD50 and RotD100 as measures of ground-motion intensity. This note provides some background that might be useful in deciding whether or not to use the new measures but makes no recommendations regarding their use.

Method

The new measures are computed in a manner almost identical to that described in Boore *et al.* (2006), except that geometric means are not computed. The two as-recorded orthogonal-component time series are combined into a single time series corresponding to an azimuth given by an increment of rotation angle using the equation

$$a_{\mathrm{ROT}}(t;\theta) = a_1(t)\cos(\theta) + a_2(t)\sin(\theta), \qquad (1)$$

where $a_1(t)$ and $a_2(t)$ are the as-recorded horizontal-component acceleration time series, and θ is the rotation angle. The response spectrum for the single time series $a_{\mathrm{ROT}}(t;\theta)$ is computed, and the process is repeated for a range of azimuths from 0° to one rotation-angle increment less than 180° (because the response spectrum is defined as the maximum of the absolute amplitude of an oscillator response, it has a rotation-angle periodicity of 180°). For the period-dependent-rotation-angle measures of motion the spectral values at each oscillator period for all values of θ are sorted, and the nnth fractiles define the measure of ground motion for that oscillator period. The computation of the period-independent-rotation-angle measures is based on minimizing a penalty function that seeks to avoid extreme variations from the desired percentile value over a broad range of periods; the algorithm for the computations is identical to that described by Boore *et al.* (2006), except that the rotation angles span a range of 180° rather than 90°.

Results

To illustrate the relations among the various measures of horizontal ground motion, Figure 1 shows a number of spectral measures obtained for the recording at the Tabas station of the 1978 Tabas, Iran, earthquake (there is no particular reason for choosing this recording, except that the filter corner frequency was low enough that usable response spectra could be obtained for periods up to about 15 sec). This figure shows that the as-recorded spectrum on the transverse component is close to the overall spectral maximum (RotD100) at long periods, whereas at shorter periods the spectrum of the longitudinal component is close to the overall maximum; RotD100 is greater than the spectrum for either as-recorded component for periods between about 1.7 and 3.5 sec. There is no reason that either as-recorded spectra should be comparable to the overall maximum spectral amplitudes for any period, but that happens to be the case for this recording. The period-independent-rotation-angle measure RotI100 is less than RotI50 for periods less than about 3.4 sec; this apparent inconsistency reveals a problem in the RotInn measures: the rotation angles for each fractile are not the same, and the spectrum computed for a single rotation angle generally will not give the maximum spectral amplitude for all periods. Because of this, the relation of the fractal nn to the actual distribution of spectral amplitudes is not obvious. Finally, engineers will probably want the true spectral maximum for each oscillator period. While RotI50 may still be an adequate measure of the median motion, the conceptual simplicity of the relation between the different fractal levels is lost for the period-independent-rotation-angle measures. For these reasons, I will emphasize the RotDnn measures in the rest of this article.

Some insight into the limitations of RotI100 can be obtained from Figure 2. The upper graph shows the rotation angles corresponding to RotD100 as a function of period,

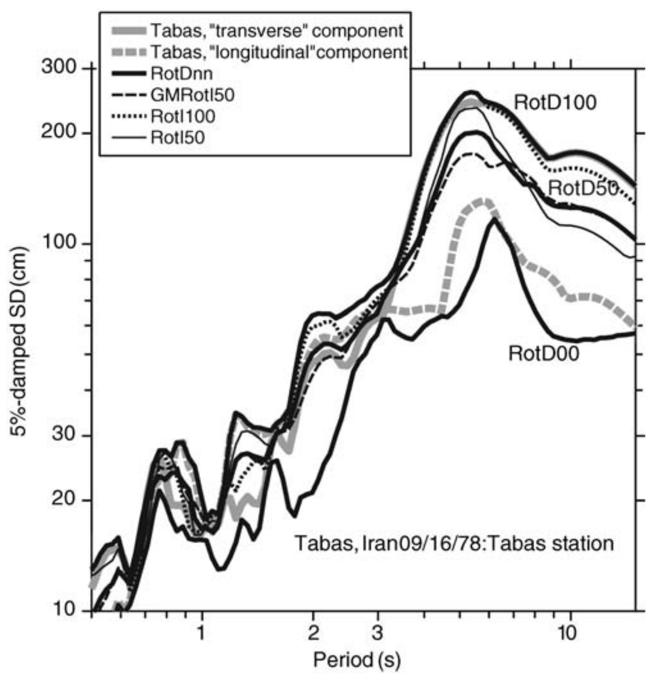

Figure 1. Various measures of 5%-damped relative response spectral displacements (SDs) for the horizontal-component recordings of the 1978 Tabas earthquake at the Tabas station. Although hard to see in the graph, RotI50 is almost equal to RotD100 for periods from 1.7 to 3.3 sec.

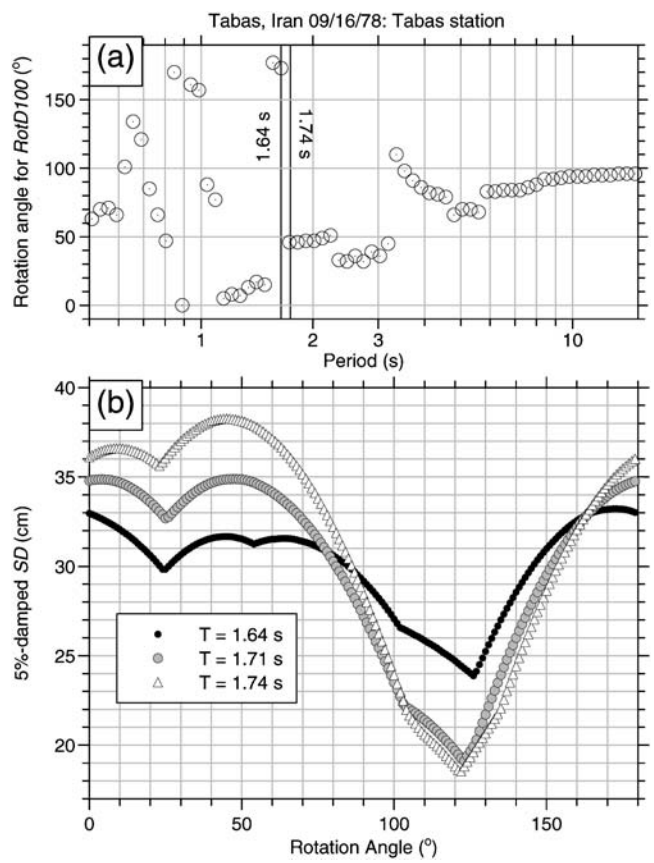

Figure 2. (a) Rotation angle corresponding to RotD100, as a function of period for the time series used in Figure 1; the two vertical lines show the periods of 1.64 and 1.74 sec for which the SDs as functions of rotation angle are shown in graph (b). The rotation angle corresponding to the maximum of SD changes from 173° to 46° for periods of 1.64 and 1.74 sec, respectively. Also shown in graph (b) is SD versus rotation angle for an intermediate period (1.71 sec) for which the two peaks in SD are about the same height (SD has a rotation-angle periodicity of 180°, and thus the peak near 180° corresponds to that near 0°).

for the time series used in Figure 1. The rotation angles stabilize to values near 90° for periods longer than about 4 sec, but for shorter periods they are quite variable and can change abruptly for a small change in period. One such change occurs near a period of 1.7 sec; the bottom graph shows why this change occurs. For a period of 1.64 sec the maximum spectral displacement (SD) corresponds to a rotation angle of 173°, and as the period increases the bump in SD near 50° becomes larger than the one near 180°, so that the rotation angle corresponding to the maximum has an abrupt shift to a smaller value. In the figure, the two bumps are about the same height for a period of 1.71 sec, but for a period of 1.74 sec the maximum corresponds to a rotation angle of 46°. The rotation angle for RotI100 is 71° (this is not shown in the figure); clearly, using this angle does not give the maximum spectral amplitude for periods less than about 4 sec, as seen in Figure 1.

To obtain statistically robust relations between the various measures, I computed the new measures of ground motion for 3225 two-component time series from the PEER NGA database (these records included the Boore and Atkinson (2008) subset of the data as well as aftershocks). Ratios of various measures of ground motion were formed, and average values and standard errors of the means of the logarithms of the ratios were computed; the antilogarithms of these means and standard errors are shown in the figures in this article. For simplicity of expression, from now on the terms "mean," "standard deviation," and "standard error" are used for the antilogarithm of the respective statistic of the logarithm of the quantity being discussed.

As mentioned earlier, the PEER NGA GMPEs used GMRotI50 as the measure of seismic intensity. To assess the impact of using two of the new measures introduced in this article, I show the ratios of RotD50 to GMRotI50 and RotI50 to GMRotI50 in Figure 3. The ratios are generally larger than unity and increase with period, reaching a value of about 1.05–1.07 at 10 sec. Thus I expect that redoing the PEER NGA equations using RotD50 or RotI50 would lead to small but systematic differences in the predicted motions. The standard deviations of the ratios are shown in the lower graph in the figure. The results in Figure 3 could be used to convert the median ground motions obtained from the PEER NGA GMPEs to the new measures of ground motion. The uncertainty in a new measure is related to the uncertainty in the previous measure, as well as the uncertainty in the ratio and the correlation coefficient between the previous measure and the ratio of the measures, as shown in equation (2) in Watson-Lamprey and Boore (2007). Using typical uncertainties and correlation coefficients in Watson-Lamprey and Boore (2007), the uncertainty in a new measure increases by less than 4% compared to the uncertainty in the previous measure.

Although the measure of median motion based on period-independent rotations (RotI50) has the advantage that it can be computed at all periods from a single time series, it also has a number of disadvantages, as discussed earlier for a single recording. Figure 4a shows the mean ratio of RotD50 to RotI50 from 3225 recordings. The ratio is period dependent but has only a 3% variation over a wide range of periods. (The source of the local peak in the ratio near 3 sec is not clear, although it may be related to the filter corner frequencies used in the record processing; a histogram of those corner frequencies, not shown here, is peaked between 2.5 and 5 sec.) The measures of median motion based on geometric means have even less variation with period, as shown in Figure 4b.

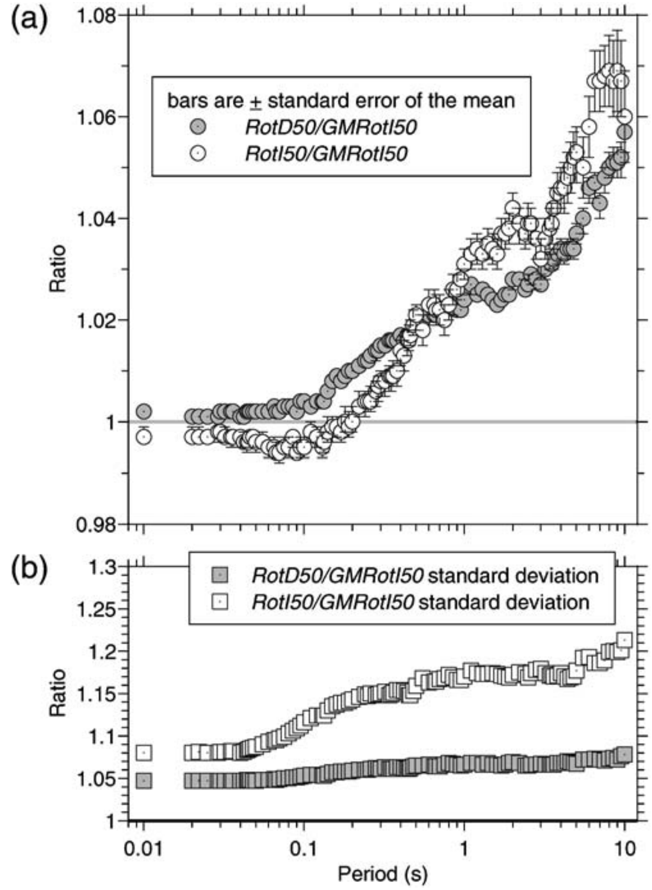

Figure 3. The antilogarithm of the mean and standard deviation of the logarithm of RotD50/GMRotI50 and RotI50/GMRotI50 for individual events as a function of oscillator period.

The relation between the measure of maximum spectral amplitude (RotD100) and GMRotI50 is shown in Figure 5. Figure 5 also shows results from regression fits to ratios derived from datasets similar to those used in this article; the Watson-Lamprey and Boore (2007) results are magnitude and distance dependent, and therefore the values shown in Figure 5 are for a representative magnitude and several distances. The information in this figure is essentially the same as contained in the Beyer and Bommer (2006) and Watson-Lamprey (2007) articles and is included for completeness.

Finally, I discuss an interesting property of the ratio RotD100/RotD50. This ratio is shown for a 1 sec oscillator for all records in Figure 6 (similar results are obtained for

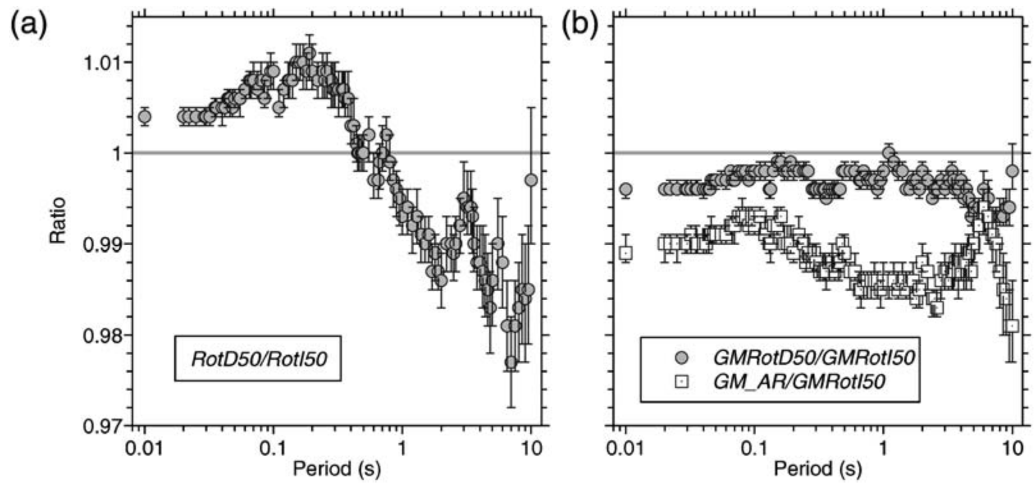

Figure 4. The antilogarithm of the mean of the logarithm of (a) RotD50/RotI50 and (b) GMRotD50/GMRotI50 and GM_AR/GMRotI50 (GM_AR is the as-recorded geometric mean) as a function of oscillator period. The bars are ± one standard error of the mean of the logarithm of the ratio.

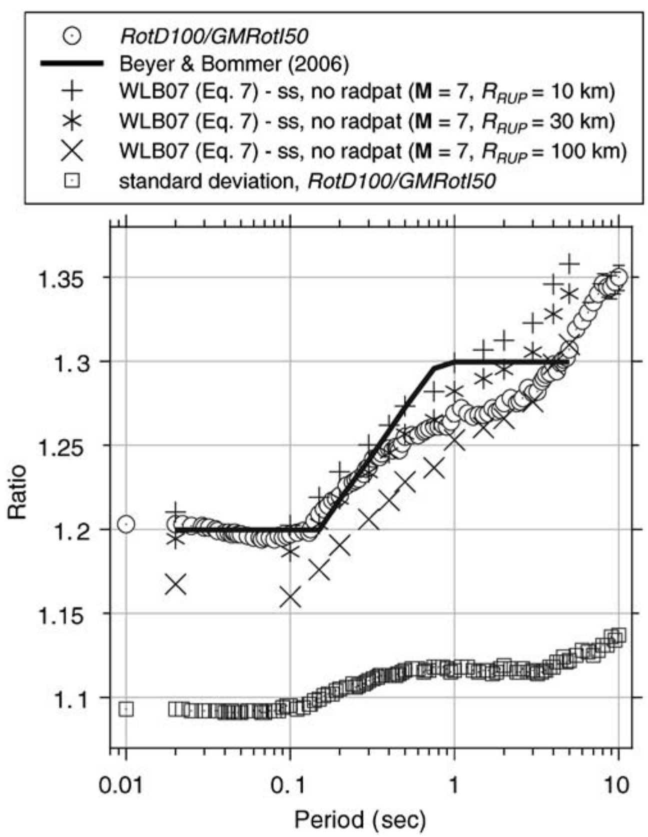

Figure 5. The relation of the period-dependent spectral accelerations to GMRotI50 (the measure used in the PEER NGA GMPEs). Also shown are the antilogarithm of the standard deviation of the logarithm of RotD100/GMRotI50 and the empirically based relations of Beyer and Bommer (2006) and Watson-Lamprey and Boore (2007) (WLB07) (the latter are magnitude and distance dependent; they are shown for one magnitude and three distances). R_{RUP} is the closest distance to the rupture surface.

other oscillator periods). Note in particular the maximum value of the ratio, which is very close to $\sqrt{2}$. This observation at first perplexed me, but it is just the theoretical ratio expected for linearly polarized ground motion. This is illustrated in Figure 7, which shows the 1.0 sec pseudo-absolute response spectral acceleration (PSA) for the Pacoima Dam recording of the 1971 San Fernando earthquake. The thick horizontal line shows the value of RotD50 (being near unity is a coincidence). Also shown is the PSA dependence on rotation angle for linearly polarized ground motion with amplitude and orientation chosen to match the RotD100 value (1.42) and rotation angle corresponding to this amplitude (30°); the theoretical dependence is given in the legend for the thin solid line in the figure. The comparison of the observed and theoretical spectral amplitudes as a function of rotation angle indicates that for this recording the

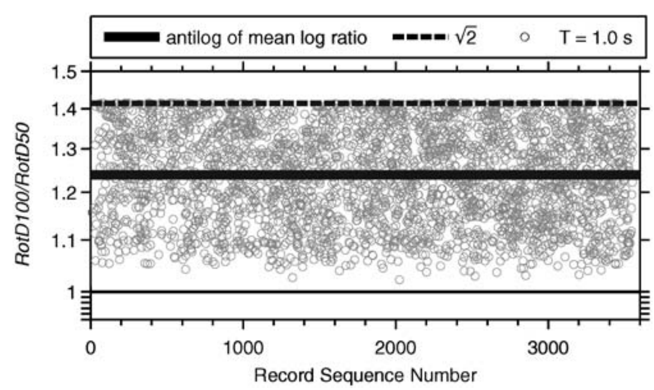

Figure 6. RotD100/RotD50 for individual events (where RotD100 is the maximum PSA over all rotation angles); the abscissa is the NGA flatfile record sequence number. The heavy black line is the antilog of the average of the logarithm of the ratio. Note that the upper limit of the ratios equals $\sqrt{2}$.

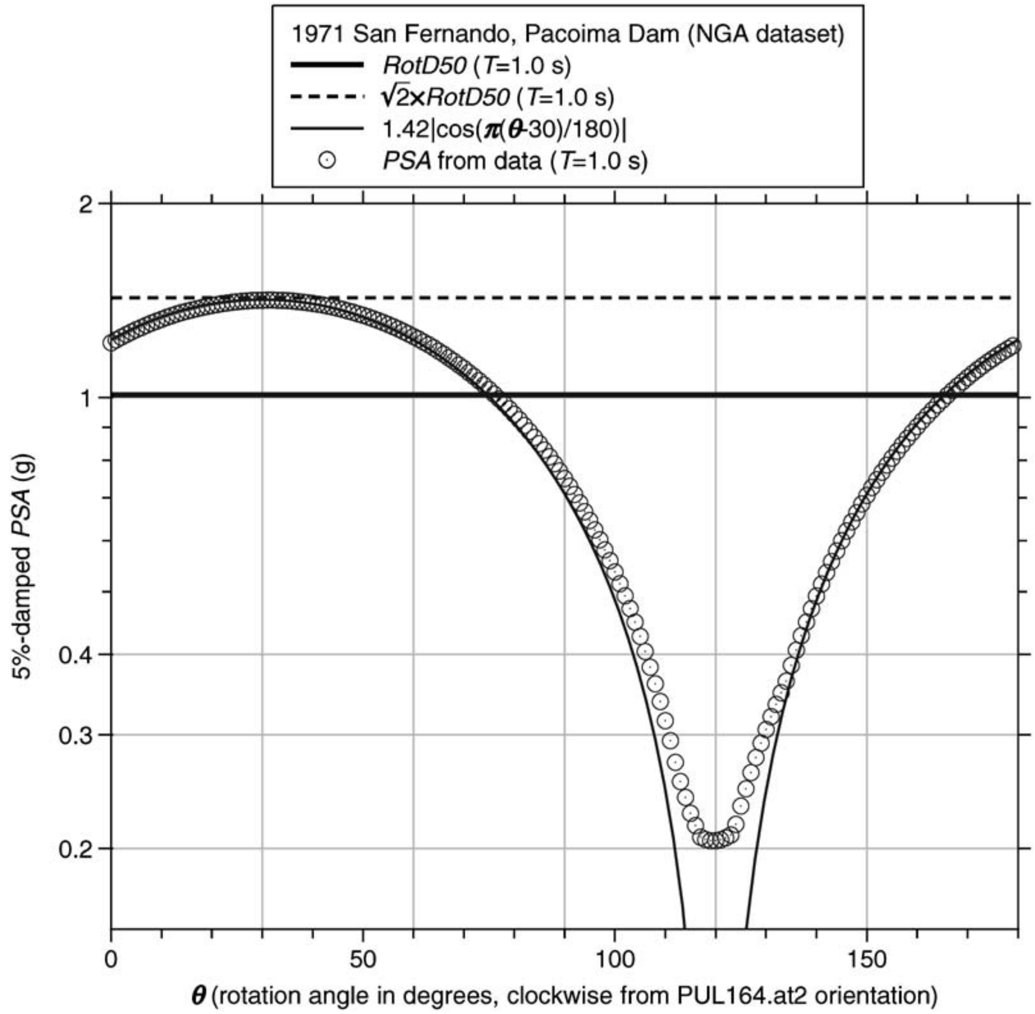

Figure 7. 5%-damped PSA for the Pacoima Dam recording of the 1971 San Fernando earthquake as a function of rotation angle for $T = 1$ sec ($\mathrm{PSA}(\theta + 180) = \mathrm{PSA}(\theta)$, so only the nonredundant rotation angles are shown). RotD50 being almost unity is a coincidence.

$T = 1$ sec oscillator response is almost linearly polarized. The upper dashed line shows the theoretical value of RotD100 for linearly polarized ground motion, given RotD50 (the theoretical value of the ratio RotD100/RotD50 equals $\sqrt{2}$).

Conclusions

This article introduces new measures of seismic intensity based on a combination of horizontal-component ground-motion recordings. The computation of these new measures is similar to that for the previously introduced geometric-mean measures GMRotI and GMRotD. Like those previous measures, the new measures are independent of the in situ orientations of the recordings; unlike the previous measures, however, no geometric means are used in the computation of the new measures. The definition of the new measures produces a logical sequence from the smallest to the largest possible spectral amplitudes over all possible ground-motion orientations; the fiftieth percentile values based on period-dependent rotation angles are similar to the fiftieth percentile values of GMRot. The measures based on period-independent rotation angles are somewhat less stable than the equivalent GMRot measures, however, and the maximum spectral amplitude from a single rotation generally does not correspond to the maximum over all periods.

Data and Resources

The time series used in computing the spectral values were provided to the developer teams of the Pacific Earthquake Engineering Research (PEER) center's Next Generation Attenuation Ground Motions (NGA) project by Y. Bozorgnia (personal comm., 2007). The time series themselves are from publicly available data centers, and the information about the record metadata and the owners of the data can be obtained from the NGA flatfile (http://peer.berkeley.edu/products/nga_flatfiles_dev.html; last accessed April 2010).

The data were reformatted using the program pea2smc.-for, and the spectra were computed using the program smc2psa_rot_gmrot.for; both of these programs are part of the TSPP package (Boore, 2010), available from the online software link on http://www.daveboore.com (last accessed April 2010).

Acknowledgments

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