# The Richter scale: its development and use for determining earthquake source parameters

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## Abstract

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The  $M_L$  scale, introduced by Richter in 1935, is the antecedent of every magnitude scale in use today. The scale is defined such that a magnitude-3 earthquake recorded on a Wood-Anderson torsion seismometer at a distance of 100 km would write a record with a peak excursion of 1 mm. To be useful, some means are needed to correct recordings to the standard distance of 100 km. Richter provides a table of correction values, which he terms  $-\log A_0$ , the latest of which is contained in his 1958 textbook. A new analysis of over 9000 readings from almost 1000 earthquakes in the southern California region was recently completed to redetermine the  $-\log A_0$  values. Although some systematic differences were found between this analysis and Richter's values (such that using Richter's values would lead to underand overestimates of  $M_{\rm L}$  at distances less than 40 km and greater than 200 km, respectively), the accuracy of his values is remarkable in view of the small number of data used in their determination. Richter's corrections for the distance attenuation of the peak amplitudes on Wood-Anderson seismographs apply only to the southern California region, of course, and should not be used in other areas without first checking to make sure that they are applicable. Often in the past this has not been done, but recently a number of papers have been published determining the corrections for other areas. If there are significant differences in the attenuation within 100 km between regions, then the definition of the magnitude at 100 km could lead to difficulty in comparing the sizes of earthquakes in various parts of the world. To alleviate this, it is proposed that the scale be defined such that a magnitude 3 corresponds to 10 mm of motion at 17 km. This is consistent both with Richter's definition of  $M_{\rm L}$  at 100 km and with the newly determined distance corrections in the southern California region.

Aside from the obvious (and original) use as a means of cataloguing earthquakes according to size,  $M_L$  has been used in predictions of ground shaking as a function of distance and magnitude; it has also been used in estimating energy and seismic moment. There is a good correlation of peak ground velocity and the peak motion on a Wood-Anderson instrument at the same location, as well as an observationally defined (and theoretically predicted) nonlinear relation between  $M_L$  and seismic moment.

An important byproduct of the establishment of the  $M_L$  scale is the continuous operation of the network of Wood-Anderson seismographs on which the scale is based. The records from these instruments can be used to make relative comparisons of amplitudes and waveforms of recent and historic earthquakes; furthermore, because of the moderate gain, the instruments can write onscale records from great earthquakes at teleseismic distances and thus can provide important information about the energy radiated from such earthquakes at frequencies where many instruments have saturated.

#### Introduction

There are numerous quantitative measures of the "size" of an earthquake, such as radiated

energy, seismic moment, or magnitude. Of these, the most commonly used is the earthquake magnitude. In its simplest form, earthquake magnitude is a number proportional to the logarithm of the peak motion of a particular seismic wave recorded on a specific instrument, after a correction is applied to correct for the change of amplitude with distance. In essence, the recorded amplitudes are reduced to hypothetical amplitudes that would have been recorded at a standard distance. The magnitude is then the difference between the distance-corrected log amplitude and a standard number used to fix the absolute value of the scale.

Because of the numerous combinations of instrument and wave types, any one earthquake can have a plethora of magnitudes assigned to it (see Båth, 1981, for a review). All magnitudes, however, stem from the pioneering work of Charles Richter (1935), who devised a scale to aid in cataloging earthquakes without reference to felt intensities. In this paper I concentrate on the Richter magnitude  $M_1$ . I review briefly the history of its development by Richter and his colleague Beno Gutenberg and then discuss some of the recent work directed at refining the magnitude scale in its type locality (Southern California) and extending its use to other areas. I close with a discussion of the relation of the  $M_{\rm L}$  scale to other measures of earthquake size and ground motion, based primarily on work done by myself and my colleagues. This paper is not intended to be a comprehensive survey of all work that has made use of  $M_{\rm L}$ .

#### History and development of the $M_{\rm L}$ scale

The idea behind earthquake magnitude is simply described in Fig. 1. Each "cloud" encloses the peak motions from a particular earthquake. Clearly, earthquake 3 is larger than earthquake 1 (and there may be little or no overlap in the distance range for which amplitudes are available; the seismograph will saturate at distances close to the large earthquake, and the signal will be below the noise level at great distances from small earthquakes). Richter credits K. Wadati with the idea of plotting peak motions against distance in order to judge the relative size of earthquakes (Richter, 1935, p. 1). If, on the average, the attenuation with distance of peak motion was the same for each event, then the vertical distance by which each cloud of points had to be moved to



Fig. 1. Cartoon showing the procedure for estimating magnitudes. Shaded areas indicate clouds of peak motion data from recordings on a particular instrument (in this case, a Wood-Anderson torsion instrument, hence "WA" on the ordinate) for individual earthquakes, the dashed line is a reference curve defined by the average attenuation of waves, and the  $M_i$  are the offset factors needed to bring each cloud of data points to the reference curve. Although the  $M_i$  represent the earthquake magnitude for each event, a plot such as this can give a misleading impression of how magnitude is defined. To avoid the possible geographic dependence of the reference curve, the Richter magnitude  $M_1$  is defined in terms of the peak motion of a Wood-Anderson instrument at a particular distance (100 km). The reference curves for different geographic regions should all pass through the same point at the defining distance (e.g., Fig. 9).

enfold, with the least mean square residual, a reference curve having the shape of the average attenuation function would be a quantitative measure of earthquake size. This distance is represented by the symbols  $M_1$ ,  $M_2$ , and  $M_3$  in Fig. 1. Formally, this can be represented by the equation:

$$M = \log A - \log A_0 \tag{1}$$

where A is the peak motion on a specific instrument and log  $A_0$  is the reference curve. Of course, both A and  $A_0$  depend on distance.

#### The original definition

Richter applied this concept to earthquakes occurring in southern California, using recordings from a network of Wood-Anderson seismographs (these instruments are simple mechanical oscillators with a natural frequency of 1.25 Hz and a design gain of 2800; they provide onscale recordings above noise for a wide range of earthquake sizes). In his landmark paper of 1935 (Richter, 1935), he determined the shape of the reference



Fig. 2. The data (symbols) used by Richter (1935) to estimate the attenuation in the southern California region. The dashed line is the average attenuation curve, the negative of which is the distance correction used to calculate magnitude. (From Richter, 1958, fig. 22-2.)

curve for correction of the measured amplitudes to a common distance by studying a small number of earthquakes occurring in January, 1932. The data and the resulting attenuation curve are given in Fig. 2, taken from Richter's textbook [Richter (1958); this curve did not appear in the 1935 paper]. The normalization Richter chose for his curve (to establish its absolute level on the ordinate) corresponds to the formal definition of  $M_{\rm L}$ . To quote Richer (1935, p. 7):

"The magnitude of any shock is taken as the logarithm of the maximum trace amplitude, expressed in microns, with which the standard short-period torsion seismometer ( $T_0 = 0.8$ sec., V = 2800, h = 0.8) would register that shock at an epicentral distance of 100 kilometers."

Note that it is this specification, and not the shape of the attenuation curve (as given, for example, by the  $-\log A_0$  values in table 22-1 of Richter's 1958 book), that corresponds to the definition of  $M_L$ . I emphasize this because determinations of  $M_L$  in regions other than southern California often use Richter's tabulated attenuation corrections, ignoring the fact that the shape of the attenuation may be regionally dependent. The proper procedure is first to determine the attenuation for each region and second to constrain the

curves at 100 km according to the formal definition of  $M_L$ . [As far as I can determine, the first published use of the symbol " $M_L$ " was not until 1956 (Gutenberg and Richter, 1956). In his original paper, Richter refers to the number obtained as the "magnitude", following a suggestion by H.O. Wood.]

In his first paper, Richter provided attenuation corrections for epicentral distances between 30 km and 600 km. Gutenberg and Richter (1942) studied data on low gain (4X) torsion instruments and published attenuation corrections for the range 0 to 25 km. The tabulated correction values were reprinted, essentially unchanged, in Richter's 1958 textbook, at which time he called the correction factor " $-\log A_0$ ". This table is reproduced in Fig. 3.

#### Synthetic Wood-Anderson seismograms

The magnitude scale was widely used in practice with apparently little or no modification for many years. Because the  $M_L$  scale has a specific instrument built into its definition, the scale was of limited use for determining magnitudes of very large or very small earthquakes, or for areas without the standard instrument within recording

	Δ (km)		∆ (km)	−log <b>A</b> ₀	∆ (km)	
	0	1.4	150	3.3	390	4.4
ſ	5	1.4	160	3.3	400	4.5
	10	1.5	170	3.4	410	4.5
Ł	15	1.6	180	3.4	420	4.5
	20	1.7	190	3.5	430	4.6
l	25	1.9	200	3.5	440	4.6
	30	2.1	210	3.6	450	4.6
ſ	35	2.3	220	3.65	460	4.6
	40	2.4	230	3.7	470	4.7
	45	2.5	240	3.7	480	4.7
	50	2.6	250	3.8	490	4.7
	55	2.7	260	3.8	500	4.7
	60	2.8	270	3.9	510	4.8
	65	2.8	280	3.9	520	4.8
1	70	2.8	290	4.0	530	4.8
	80	2.9	300	4.0	540	4.8
	85	2.9	310	4.1	550	4.8
	90	3.0	320	4.1	560	4.9
	95	3.0	330	4.2	<b>57</b> 0	4.9
	100	3.0	340	4.2	580	4.9
1	110	3.1	350	4.3	590	4.9
L	120	3.1	360	4.3	600	4.9
	130	3.2	370	4.3		

Fig. 3. The distance correction table in Richter (1958, table 22-1), which is based on Richter (1935) at distances beyond 30 km and Gutenberg and Richter (1942) at closer distances.



Fig. 4. Recorded accelerogram and computer-derived Wood-Anderson seismogram (note the scale in meters). (From Kanamori and Jennings, 1978.)

range. With the advent of digital recordings, however, it became possible to simulate the response of a standard Wood-Anderson seismometer and thus overcome the limitations of limited dynamic range or lack of the standard instrument. Among the first to construct synthetic Wood-Anderson records were Bakun and Lindh (1977), who calculated magnitudes for small earthquakes (down to

records were Bakun and Lindh (1977), who calculated magnitudes for small earthquakes (down to  $M_{\rm L} = 0.1$ ). Kanamori and Jennings (1978) used accelerometer recordings to compute synthetic Wood-Anderson seismograms for moderate to large earthquakes at distances for which the standard Wood-Anderson instrument would be driven offscale. Figure 4 is an example of their calculation for a station about 30 km from an earthquake with moment magnitude of 6.6. Note that the peak amplitude of the synthetic Wood-Anderson trace is about 15 *meters*, thus emphasizing the limitation of the standard instrument.

An implicit assumption in comparing the  $M_L$  calculated from simulated and real Wood-Anderson instruments is that the instrument response of the actual Wood-Anderson seismograph is adequately represented in the simulations. Unfortunately, there is evidence from New Zealand, Australia, and the University of California–Berkeley network that the effective magnification of Wood-Anderson instruments may be about 0.7 of the design value of 2800 (B.A. Bolt and T.V. McEvilly, oral commun., 1984; D. Denham, written commun., 1988). Station corrections derived from regression analyses can compensate for differences between real and assumed magnifications, if multiple recordings are available at any



Fig. 5.  $M_L$  residuals (estimates from each recording minus the mean magnitude for each event) as a function of distance. The top panel shows the results of Jennings and Kanamori (1983) and represents the average of a number of recordings, the middle panel includes data from Italian earthquakes studied by Bonamassa and Rovelli (1986), and the bottom panel, from Hadley et al. (1982), shows  $M_L$  computed using theoretical calculations of peak motion and Richter's  $-\log A_0$  correction. (From Bonamassa and Rovelli, 1986.)

one site and any one earthquake is recorded on both accelerographs and real Wood-Anderson instruments. If  $M_L$  were only determined from actual Wood-Anderson instruments and if the magnification of all the instruments differed by the same amount from the design magnification, then the magnitude determinations would be internally consistent. Neither of these conditions are true, however, and therefore biases and scatter are introduced into estimates of  $M_L$ . It would be useful for all observatories running Wood-Anderson seismographs to do careful calibrations of the instruments.

# Modifications to $-\log A_0$ in southern California

Even though Richter (1935) and Gutenberg and Richter (1942) used very few events in their determination of the log  $A_0$  corrections, the adequacy of the correction was, of course, subjected to continuing implicit evaluation in the course of routine determinations of magnitudes by personnel at the Seismological Laboratory of the California Institute of Technology (Caltech). It became apparent the magnitudes were being underand over-estimated for close and distant earthquakes, respectively, relative to earthquakes at distances near 100 km. The most straightforward explanation for these systematic effects was that as averaged over many events and much of southern California, the actual attenuation of peak motions from Wood-Anderson amplitudes was slightly different than implied by the log  $A_0$  corrections.

The bias at close distances has been shown by many studies, both in California and in other countries (Luco, 1982; Jennings and Kanamori, 1983; Bakun and Joyner, 1984; Fujino and Inoue, written commun., 1985; Bonamassa and Rovelli, 1986; Greenhalgh and Singh, 1986; Hutton and Boore, 1987). A few of these findings are summarized in Fig. 5, which shows residuals of the  $M_L$  determined from individual stations relative to the average value for each event, as a function of distance. It is clear that individual  $M_L$  estimated at distances between about 10 to 40 km are low relative to the average  $M_L$ . Also shown in Fig. 5 are residuals from some theoretical calculations by

Hadley et al. (1982) that are in agreement with the observed residuals. As Hutton and Boore (1987) point out, the rather ubiquitous bias has a simple explanation: Gutenberg and Richter (1942) based their log  $A_0$  values on residuals of data (none closer than 22 km) as a function of epicentral distance, relative to the attenuation expected for an average focal depth and a specified geometrical spreading. Unfortunately, they used incorrect values for the effective geometrical spreading  $(1/r^2)$ and the focal depth (18 km). These two factors trade off such that calculations of  $M_{\rm L}$  at distances of about 10 to 40 km are systematically low relative to the magnitudes calculated for appropriate geometrical spreading (close to 1/r) and more accurately determined focal depths (generally shallower than 18 km).

In view of the bias in Richter's  $-\log A_0$  corrections found both by studies of simulated Wood-Anderson records and in the course of routine analysis at Caltech, Kate Hutton and I undertook a formal analysis of Wood-Anderson data from southern California with the goal of improving the distance correction used in the determination of  $M_L$  (Hutton and Boore, 1987). We applied straightforward regression analysis to 7355 measurements from 814 earthquakes (Fig. 6) to derive station corrections and the following equation for the distance correction:

$$-\log A_0 = 1.110 \log(r/100) + 0.00189(r - 100) + 3.0$$
(2)

where r is the hypocentral distance in kilometers and the peak motion from Wood-Anderson seismographs is in millimeters. This correction curve is compared to Richter's  $-\log A_0$  values in Fig. 7. In view of the limited data used by Richter, the agreement between the new findings and Richter's values is impressive. There are, however, systematic differences of up to 0.4 magnitude units between the corrections. The bias for distances between 10 and 40 km.discussed in the preceding paragraph is clearly seen (in the inset of the figure), as is a bias at distances beyond 200 km. This latter bias is important, for most data for earthquakes larger than about  $M_L$  4 come from distances greater than 200 km. The nature of the



Fig. 6. Map of southern and central California, showing events and stations used in Hutton and Boore's analysis. The box in the upper central portion of the top panel encloses earthquakes near Mammoth Lakes. An unusually large number of earthquakes occurred in this region, and Hutton and Boore were concerned that the results might be biased by the unweighted inclusion of these data. (From Hutton and Boore, 1987.)



Epicentral Distance (km)

Fig. 7. The  $-\log A_0$  curve from Hutton and Boore (1987) for focal depths of 0, 10, and 20 km, with and without the Mammoth Lakes data (dashed and solid lines, respectively). Richter's values are shown for comparison. (From Hutton and Boore, 1987.)

biases are consistent with those noticed during routine magnitude determination: the magnitudes are under- and over-estimated for close and distant events. Some sense for the degree of the bias can be obtained from Fig. 8, which shows the magnitude residuals, relative to the "official" Caltech  $M_{\rm L}$  value, for the events used in the regression study. The top panel shows residuals of the  $M_{\rm L}$  values determined from Hutton and Boore's correction factors  $[M_{\rm L}({\rm HB})]$  with respect to the Caltech values  $[M_{\rm L}({\rm CIT})]$ . In analyzing this figure, the correlation between size of the earthquake and the distance at which onscale recordings above the noise level obtained must be kept in mind. Thus the small earthquakes have many recordings within 40 km,  $M_{\rm I}$  (CIT) is therefore systematically lower than  $M_{\rm L}$  (HB), and the residual is positive. The same analysis applied to the larger events predicts negative residuals. For events greater than  $M_{\rm L} \approx 5.3$ , however, the explanation of the bias is more complicated. This is best shown in the bottom panel, which shows the residuals of  $M_1$  computed using Richter's  $-\log A_0$ corrections  $(M_{\rm L}({\rm R}))$  and the same data and station corrections used for the  $M_{\rm L}$  estimations in the top panel. For the smaller earthquakes, the residuals scatter about zero, as they should (this is a consistency check on  $M_{\rm L}$  (CIT)). In contrast to the small events, however, the bias for the large events has been only slightly reduced. This inconsistency may be due to a number of things, mostly stemming from the fact that the larger events will saturate most of the standard Wood-Anderson instruments in the southern California region. These factors include different station corrections used for the low-gain "100X" torsion instruments, the use of data outside the southern California area, and subjective judgement on the part of the analysts assigning magnitudes. A careful study of the magnitude assignment [both  $M_1$  (CIT) and  $M_{\rm L}({\rm HB})$ ] for each of the larger earthquakes remains to be done.



Fig. 8. Difference between recomputed magnitudes and those in the California Institute of Technology catalog for the same event. The upper figure used the attenuation correction given by Hutton and Boore (1987) and the lower used Richter's log  $A_0$  correction. The slanting line on the right side of the figure has a slope of -0.3 (as found by Luco, 1982, in his relation between  $M_L$  found from simulated Wood-Anderson recordings and those determined at greater distances from actual recordings). (From Hutton and Boore, 1987).

## Attenuation corrections in other regions

As earlier emphasized, the  $-\log A_0$  corrections should be regionally dependent, except at 100 km (where the scale is defined). Richter recognized this, saying:

"Another limitation of Table 22-1 [Fig. 3 in the present paper] is that without further evidence it cannot be assumed to apply outside the California area" [Richter, 1958, p. 345].

Although the  $M_L$  scale has been in existence for more than 50 years, to my knowledge there have been few published studies of attenuation corrections, and those that have appeared were published within the last 5 years. I have summarized a number of these in Fig. 9, which shows the difference between the  $-\log A_0$  corrections in southern California and in other regions as a function of distance. Generally, within 100 km the curves are similar, with a large divergence beyond that distance. This is not surprising, for anelastic attenuation and wave propagation effects in differing crustal structures should not play a large role at the closer distances.

The determination of regionally-dependent attenuation corrections does not guarantee uniformity of the  $M_L$  scale; site response and station



Fig. 9. Differences in  $-\log A_0$  corrections for various parts of the world, relative to Hutton and Boore's (1987) finding for southern California. Central California (Bakun and Joyner, 1984); central Great Basin, western United States (Chavez and Priestley, 1985); southern Great Basin, western United States (Rogers et al., 1987); Greece (Kiratzi and Papazachos, 1984); South Australia (Greenhalgh and Singh, 1986); Japan (Y.

Fujino and R. Inoue, written commun., 1985).

corrections must also be considered. Unfortunately, neither of these considerations are included in the formal definition of  $M_1$ . Consider two networks, both located in regions with identical crustal structure below several kilometers. Let all the stations in one network be sited on harder rock, with less increase in seismic velocity as a function of depth in the upper several kilometers, than stations in another network. Amplification of seismic waves near the stations will be different for the two networks, leading to differences in the recorded amplitudes of earthquakes, everything else being the same. In general, the stations on harder rock will record smaller amplitudes than those on the softer material, and this could lead to a bias in  $M_{\rm L}$  of several tenths of a magnitude unit. Station corrections will not correct for this internetwork bias, for they are intended to account for systematic variability within a network.

It is clear from routine assignment of magnitude at seismological observatories that station corrections are needed to account for systematic variations in the  $M_L$  derived from various stations within a network. These corrections can be

surprisingly large (at least 0.3 units for southern California), even for stations sited on firm material. As the corrections are relative, it is necessary to specify a constraint that must be satisfied by the corrections. Without a constraint, any number could be added to all the corrections. The corrections would still account for interstation variability, but obviously the absolute value of the magnitudes would be meaningless; for global uniformity of the  $M_1$  scale, an additional statement regarding the average site conditions needs to be added to Richter's definition. The constraint on the station corrections might be that they all add to zero, or that one station has a fixed correction. The latter is preferred if one station has a particularly complete recording history, for then changes in the station distribution over time will not lead to a new set of station corrections for all stations (this can lead to apparent time-dependent changes in seismicity).

The issues of regional variations in attenuation corrections, as well as appropriate station corrections, have been much discussed in the nuclear-discrimination field. Although usually concerned with magnitudes other than  $M_L$ , these studies have much to offer. The very comprehensive review of magnitude scales by Båth (1981) contains a number of relevant references.

## A new definition of $M_{\rm L}$

Defining the  $M_1$  scale at 100 km can lead to misleading comparisons of the size of earthquakes if the attenuation of seismic waves within 100 km is strongly dependent on region. For example, for an earthquake with a fixed  $M_1$ , the amplitude on a Wood-Anderson instrument at 15 km predicted by the Chavez and Priestley (1985)  $-\log A_0$  relation for the Great Basin of the western United States is more than twice that predicted by the attenuation correction for southern California (Hutton and Boore, 1987). Although Fig. 9 suggests that this is an extreme case, the difficulty could be avoided by defining the  $M_{\rm L}$  scale at a closer distance. In our paper, Kate Hutton and I have proposed that the magnitude be defined such that a magnitude 3 earthquake corresponds to a 10 mm peak motion of a standard Wood-Anderson instrument at a hypocentral distance of 17 km. We chose 17 km so that the peak motion would be a round number and because uncertainties in source depth and the problems associated with finite rupture extent for the large earthquakes would not be as important as they would be for a definition based on a closer distance.

# The uses of $M_{\rm L}$

I conclude this paper with a short discussion of some uses of  $M_L$ , particularly as they relate to the estimation of source size and the prediction of ground motion. Of course, the main use of the  $M_L$ scale has been in providing a simple, quantitative measure of the relative size of earthquakes. Catalogs of earthquake occurrence can then be ordered according to  $M_L$ , and statistics related to earthquake occurrence can be computed. This use of  $M_L$  is well known, and I will say nothing more about it.

## The relation between seismic moment and $M_L$

The seismic moment of an earthquake  $(M_0)$  is widely recognized as the best single measure of the "size" of an earthquake. There have been numerous attempts to relate  $M_0$  to various magnitude measures. If these correlations are made for magnitudes sensitive to different frequency ranges of the seismic waves, they provide useful information on the systematic behavior of the scaling of the energy radiation from the seismic source. The correlation of  $M_0$  and  $M_L$  has been made in many studies. These correlations usually take the form:

$$\log M_0 = cM_{\rm L} + d \tag{3}$$

There have been some apparent inconsistencies in the published c coefficient, even for earthquakes in a single geographic area. Bakun (1984) and Hanks and Boore (1984) showed that these differences in c are largely a result of the different  $M_0$  ranges used in the various studies. In general, c for the larger earthquakes is greater than for the small events. Furthermore, this difference is expected on theoretical grounds. It is the consequence of the interaction of the corner frequency of the earthquake with the passband formed by



Fig. 10. Moment-magnitude data for central California earthquakes (pluses; box for 1906 earthquake encloses various estimates; see Hanks and Boore, 1984, fig. 2, for sources of data). The theoretical calculation uses the stochastic model described by Boore (1983, 1986b), with source scaling according to Joyner (1984), a stress parameter of 50 bars,  $\kappa = 0.03$ , frequency-dependent amplification to account for the reduced velocities near Earth's surface (Boore, 1986b, table 3), 1/rgeometrical spreading, and frequency dependent Q. The calculation was done at 100 km.

the instrument response and the attenuation in the earth. To illustrate this point, Fig. 10 shows  $M_0$ and  $M_{\rm L}$  data from central California, along with theoretical predictions (in this and the next two figures, the theoretical predictions have been made using the same model for the source excitation, attenuation, and site amplification; see the figure caption for details). The data have not been corrected for the biases discussed earlier, or for biases due to site amplification that might exist in the determination of the moment of the small earthquakes (Boore, 1986a, p. 281). Doing so would tend to improve the fit of the data to the theoretical prediction (which was not adjusted to fit the data). Even without the correction, the change in slope is obvious in the data.

# Predictions of ground motion in terms of $M_{\rm L}$

Because  $M_L$  is determined by waves with periods in the range of the resonant periods of common structures, it is potentially a useful magni-



Fig. 11. Peak Wood-Anderson response and peak velocity calculated at distances of 10, 50 and 200 km, using the stochastic model described in the caption to Fig. 10. At each distance, the calculations were made for moment magnitudes ranging in 0.5 unit steps from magnitude 3 to magnitude 8. The heavy dashed line is the relation found from data (Boore, 1983, figs. 8 and 9). A similar plot for the response of a seismoscope is given in Boore (1984, fig. 5).

tude measure for engineering practice. In particular,  $M_L$  should be closely related to the ground motion parameters that engineers use in the design of structures. For example, both data and theory show a strong correlation between the Wood-Anderson peak amplitude and the peak ground velocity at the same site (Boore, 1983, figs. 8 and 9). The relation derived from the data is given by:

$$pgv = 0.0077wa \tag{4}$$

where pgv and wa are the peak velocity and Wood-Anderson amplitudes in cm. This relation is shown by the dashed line in Fig. 11. Also shown in the figure are theoretical amplitudes for a range of moment magnitudes at distances of 10, 50, and 200 km. The theoretical calculations show that although the relation between pgv and wa is multivalued, depending on distance, the relation given by eqn. (4) approximates an envelope to the theoretical curves (the divergence of the theoretical curves from the dashed line is primarily for small and large earthquakes, which are poorly represented in the data set leading to eqn. (4).

Another way of observing the relation of  $M_L$  to ground motion parameters is in the scaling of

peak motion with magnitude at a fixed distance. An example, for peak acceleration and peak velocity, is shown in Fig. 12. In that figure, theoretical estimates of the peak motions have been made at 20 km for moment magnitudes (M) ranging from 3 to 7.  $M_L$  was determined from theoretical peak motions of a Wood-Anderson instrument at 100 km (as in Fig. 10). The solid and dashed lines show the scaling of the peak acceleration and peak velocity as a function of M and  $M_L$ , respectively. The dependence on M shows a change in slope, particularly for peak acceleration, whereas the scaling with  $M_L$  is more nearly linear. In effect,



Fig. 12. Scaling of peak velocity and peak acceleration as a function of moment magnitude M (solid line) and Richter local magnitude  $M_L$  (dashed line), using the stochastic model described in the caption to Fig. 10. The calculations were made at a distance of 20 km for moment magnitudes ranging from 3.0 to 7.0. The dashed line shows the  $M_L$  dependence of the ground motion for the same range of moments as used to construct the solid line.

similar, leading to a linear relation between peak

ground motion and  $M_{\rm L}$ . Because of the correlation of ground motion with  $M_{\rm L}$  and because many seismicity catalogs are in terms of  $M_{\rm L}$ , it would seem natural to use  $M_{\rm L}$ as the predictor variable for magnitude in equations relating ground motion to distance and magnitude. This may not be wise, however, mainly because of the difficulty of predicting  $M_{\rm L}$  for design earthquakes.  $M_L$  for such events is often estimated from magnitude-frequency statistics derived from catalogs, but there is the problem that  $M_{\rm L}$  values in these catalogs for large earthquakes are often poorly determined or of uncertain meaning (e.g., Fig. 8). A better magnitude to use for estimation of ground motions is the moment magnitude, M. It is directly related to the overall size of earthquakes, geologic data (slip rate information, fault extent, etc.) can be used to estimate the magnitude of future earthquakes, and both theoretical (Boore, 1983) and observational studies (Joyner and Boore, 1981, 1982) find a good correlation between ground motion parameters and M. It has the further advantage that ground motion parameters are less sensitive to errors in M than in  $M_{\rm L}$  (compare the slopes of the solid and dashed curves in Fig. 12).

#### Conclusions

The Richter scale is a well established means of assigning a quantitative measure of size to an earthquake. Recent work has focused on obtaining the regionally dependent distance corrections needed to estimate  $M_{\rm L}$ ; in southern California, Hutton and Boore (1987) find that Richter's  $-\log A_0$  values lead to somewhat biased estimated of magnitude (in view of the very small number of events used to establish his correction factors, however, Richter's results are surprisingly good).  $M_{\rm L}$  and seismic moment are correlated, although both data and theory show that the correlation is not linear. The peak motions on Wood-Anderson seismographs are proportional to the peak velocity of the ground motion at the same site, although the correlation depends on distance. When used as the explanatory variable in the scaling of peak ground motion as a function of magnitude at a fixed distance,  $M_L$  leads to a more linear (and steeper) relation than is obtained using moment magnitude.

An important byproduct of the establishment of the  $M_{\rm L}$  scale is the continuous operation of the network of Wood-Anderson seismographs on which the scale is based. In southern California, 6 decades of records are available from the same instruments. This has proven useful in comparing recent and historic earthquakes. For example, Bakun and McEvilly (1979) used such records to demonstrate that the 1934 sequence of earthquakes near Parkfield, along the San Andreas Fault, was very similar to that occurring in 1966. This finding gives support for the concept of characteristic earthquakes in this area and is an important component of the prediction of a similar sequence in the late 1980's and early 1990's. The records from Wood-Anderson seismographs are also a source of onscale recordings at teleseismic distances from great earthquakes. Such earthquakes commonly saturate instruments intended for routine recording at teleseismic distances. As an example, Hartzell and Heaton (1985) used Wood-Anderson records from the 1964 Alaska earthquake in their study of source time functions of large subduction zone earthquakes. In view of these unconventional, but important, uses of the Wood-Anderson seismographs, as well as the need to maintain continuity and uniformity in the magnitudes reported for earthquakes, it is important to continue the operation of the instruments.

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