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Comments on 'Scattered surface waves from a surface obstacle' by J. A. Hudson

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1 Introduction

The paper 'Scattered surface waves from a surface obstacle', published in 1967 (Hudson 1967), contains a theoretical development of the scattering of Rayleigh waves from a region of uneven topography or from a surface inclusion. The slopes of the surface and of the boundary of the inclusion are assumed to be small, and so are the ratios of the elevation of the surface and the thickness of the inclusion to the wavelength of the disturbance. The theory is in fact a straightforward extension to a two-dimensional surface of the method of Gilbert & Knopoff (1960); the unevenness of the surface is replaced by approximately equivalent surface loads.

The results of this analysis were used in the latter part of the paper to interpret data which had been collected by Key (1967) and which showed *P* arrivals at the Eskdalemuir array followed closely by short-period Rayleigh waves. These surface waves appeared to be the result of the scattering of the incident *P* wave at a nearby deep valley known as Moffatt Water. Comparison of theoretical and observed ratios of *P* to Rayleigh wave spectra gave the reasonable result that the scatterer was approximately 5 km wide.

Unfortunately, the paper is marred by three separate errors (only one of which, however, affects the main conclusion) and it seems that, in spite of the long time which has elapsed since its publication, it would be useful to expose and correct these errors here.

The ground motion associated with short-period surface waves, strongly guided by sedimentary layers, is of great interest for seismic engineers (see, for instance, Swanger & Boore 1978a,b), and the generation of such waves by uneven topography and by heterogeneous geological structure is quite efficient; the amplitude of the scattered wave from Moffatt Water, for instance, is up to 40 per cent of the surface amplitude of *P*. Techniques for prediction of the characteristics of such scattered waves are very few apart from the perturbation method (briefly described above) and a straight numerical approach. It is most important to know, therefore, to what extent the results of perturbation theory are accurate.

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The relief map of Moffatt Water (Fig. 6 of Hudson 1967) shows that an approximate value for the depth of the valley is 0.4 km, and that an estimate of 8 km for its half-length is reasonable. It implies, however, that the angle subtended by the scatterer at the receiver is not small as assumed in the derivation of the theoretical result; the half-angle is $\zeta \cong \tan^{-1}(8/13) \simeq 0.6$ (30°). Therefore, we have neglected terms of order $\frac{1}{2}\zeta^2 \simeq 0.2$ which, if included, would have changed the estimates of b by up to about 20 per cent.

The values of a, b, and h are rather approximate but they are nevertheless encouraging with regard to the use of perturbation theory in this type of problem, for which the slopes (about 30°) are not small. It is perhaps worth mentioning here that two different model studies (Hudson *et al.* 1973; Ipatova & Rykunov 1974) have shown that perturbation theory gives good results for two-dimensional scattering of plane waves to Rayleigh waves so long as the slope of the surface is not too steep and the extent of the roughness not too great. Hudson *et al.*, for instance, obtain good correlation between theory and experiment for a single surface groove with a slope up to 25° and for up to four parallel grooves with slopes of 10°. (This is with a groove width equal to a Rayleigh wavelength.) Ipatova & Rykunov find good correlation between experimental and theoretical results when the height of the surface irregularity is less than a tenth of a wavelength.

4 Scattering from an incident S wave

For applications in engineering seismology, it is more interesting to be able to predict the amplitudes of surface motion from an incident S wave than from a P wave; the S wave carries more energy and is, in other circumstances, a more efficient scatterer (see, for instance, Knopoff & Hudson 1964).

Equation (1) is still applicable, but with different expressions for the $T_{xx}, T_{xy}, \ldots, T'_{zz}$ from those given by equation (4.5) of Hudson (1967). With normal incidence, the unperturbed wave is

$$\bar{\mathbf{u}} = \frac{2\omega\bar{F}(\omega)}{\beta}(\cos\theta, \sin\theta, 0)\cos(\omega z/\beta),$$

where θ is the polarization angle, and

$$T'_{xz} = -2(\alpha/\beta)^3\cos\theta$$

$$T'_{yz} = -2(\alpha/\beta)^3\sin\theta.$$

The other coefficients are all zero.

Only T'_{xz} is relevant to the generation of Rayleigh waves in the x-direction; that is, only waves polarized in the x-direction contribute to the scattered signal in this case owing to the symmetry of the function g. Substituting T'_{xz} into equation (1) we get

$$\bar{u}_z^R \triangleq \frac{i\exp(i\pi/4)\,\omega^{7/2}\bar{F}(\omega)\,\gamma^{3/2}(2/\gamma^2 - 1/\beta^2)\cos\theta\,\bar{g}_1}{4(2\pi R)^{1/2}\beta^5\Delta}$$

and the spectral ratio of the amplitude of the scattered Rayleigh to that of the shear wave polarized in the x-direction is

$$S^{SR} = \left|\frac{\gamma^{3/2}(2/\gamma^2 - 1/\beta^2)}{8\beta^4\Delta}\right|\left(\frac{\pi}{2R}\right)^{1/2} hab\,\omega^{5/2}\exp[-(\omega a/4\gamma)^2]. \tag{6}$$

The ratio of this quantity to the equivalent expression (equation 4) for an incident P wave is

$$\frac{S^{SR}}{S^{PR}} = \frac{2\nu'}{\gamma(2/\gamma^2 - 1/\beta^2)}, \tag{7}$$

which, with the above values of γ and β gives

$S^{SR}/S^{PR} = 0.66.$

The P wave is in fact a more efficient scatterer than the S wave in this case.

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