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Effect of Higher Mode Contamination on Measured Love Wave Phase Velocities

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In some circumstances the first higher mode of the Love surface wave can be significantly excited in the period range 30–90 sec by earthquakes and may travel at a group velocity comparable to that of the fundamental mode. This difficult-to-separate contamination will produce a large scatter, but no uniform bias, in phase velocities measured from an ensemble of events. The phase velocities measured from a single event may show a bias over a limited range of periods, but this perturbation may be positive or negative. These results argue against an explanation of observed inconsistencies of Love and Rayleigh wave phase velocities as being due to higher mode contamination. The results are also applicable to the effect of multipath interference of two similar-mode waves traveling at an angle with respect to one another.

INTRODUCTION

At the recent American Geophysical Union meeting in Washington, *Thatcher and Brune* [1969] pointed out the importance of the contamination of the fundamental Love surface wave mode by the first higher mode and speculated that it could explain observed inconsistencies between Love and Rayleigh wave phase velocities [see, e.g., *Aki and Kaminuma,* 1963; *McEvilly,* 1964]. The first higher Love mode can have, at periods of 30–90 sec, group velocities very similar to the fundamental mode and furthermore can be significantly excited by earthquakes. The equality of the group velocity means that it would be nearly impossible to separate the two modes on seismograms. The purpose of this note is to discuss the effect of this higher mode contamination on phase velocities measured between two in-line stations. The results indicate that the presence of the higher mode can produce significant scatter in the measured phase velocities, but that no consistent bias should result if a number of events are used.

FORMULATION AND SOLUTION

This paper is basically concerned with the perturbation in measured phase velocity arising from the interference of the desired mode and another mode having different dispersion characteristics. Such interference in general is not a problem, for the group velocity curves of the two modes are usually dissimilar enough so that the presence of an interfering mode is obvious on the record and can be eliminated by appropriate group velocity filtering. For Love waves, however, the fundamental and first higher mode group velocity curves can approach one another in the period range 30–90 sec (Figure 1). Furthermore, the relative excitation of the first higher mode to the fundamental mode can be quite large for typical earthquakes (Table 1). Since the period range affected is essentially that used in studies of lower-crust upper-mantle structure, the effect of the interfering mode is thus very important. Figure 1 and Table 1 imply that earthquakes within ocean basins and having predominantly oceanic propagation paths to a given station should be most effective in producing higher mode contamination. The figure and table are based on idealized oceanic and continental models, however, and the results, being strongly influenced by the structure of the low velocity zone, might change if other models were used. Thatcher (personal communication, 1969) mentioned that a wide range of continental and oceanic models gave group velocities and relative excitations appropriate to the production of significant higher mode interference. Thus I will assume that a contaminating mode is present and proceed to discuss its effects on the measured phase velocity.

Consider two stations at distances x_A and x_B along a great circle path from the epicenter. Let the distance between the two stations be δx. The total motion at station A can be represented by the Fourier integral

$$u_T(x_A, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{1 + \eta e^{i\epsilon}\} A_0 \cdot \exp[\phi_0 + \omega t]\, d\omega \qquad (1)$$

and at station B by:

$$u_T(x_B, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{1 + \eta e^{i(\epsilon+\gamma)}\} A_0 \cdot \exp[-k_0\cdot\delta x + \phi_0 + \omega t]\, d\omega \qquad (2)$$

where subscripts 0 and 1 refer to fundamental and first higher modes respectively, $\eta = A_1/A_0$, $\epsilon = \phi_1 - \phi_0$, $\gamma = \delta k\cdot\delta x$, and $\delta k = k_0 - k_1$, where k_i is the wave number for a given mode. A_i and ϕ_i are the amplitude and phase spectrums of the individual modes at station A. The ϕ_i include a source term, a term due to epicenter-station propagation, and a fiducial term arising from the choice of an arbitrary time origin. We assume that the same time origin is used at stations A and B. This assumption is of no consequence in the results.

By writing the first bracketed term in each integral in terms of an amplitude and phase, we can write the phase difference between the two stations as

$$\Delta\phi = \{\phi_0 + \phi_A^*\} - \{\phi_B^* - k_0\cdot\delta x + \phi_0\}$$

or

$$\Delta\phi = \phi_A^* - \phi_B^* + k_0\cdot\delta x \qquad (3)$$

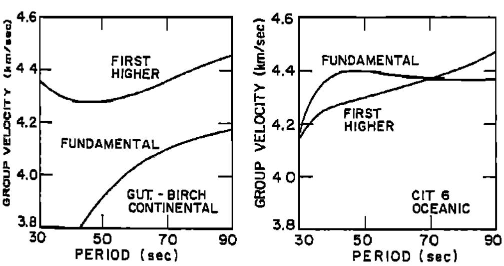

Fig. 1. Group velocity curves for the fundamental and first higher Love modes for typical continental and oceanic structures [from *Anderson and Toksöz*, 1963].

TABLE 1. Relative Surface Displacement of First Higher Love Mode Compared with Fundamental Love Mode for a Point Force Source at the Surface and at a Depth of 100 km

Surface focus			Source at 100 km		
T, sec	Shield	Oceanic	*T*, sec	Shield	Oceanic
20	0.00018	1.94	20	0.0065	42.0
40	0.093	0.95	60	0.0	0 4
60	0.364	0.93			
80	0.396	0.68			

Note: These values are approximate and were calculated from values given in *Harkrider and Anderson* [1966] and *Anderson and Toksöz* [1963] by using formulas given in *Saito* [1967].

where ϕ_i^* is the phase perturbation at each station due to the higher mode contamination. These terms are given by

$$\phi_A^* = \tan^{-1}[\eta \sin\epsilon/(1 + \eta\cos\epsilon)] \qquad (4)$$

$$\phi_B^* = \tan^{-1}[\eta\sin(\epsilon+\gamma)/(1 + \eta\cos(\epsilon+\gamma))] \qquad (5)$$

Finally, we define $\delta\phi = \phi_A^* - \phi_B^*$ as being between -0.5 and 0.5 circles. Then we can write (3) as

$$\Delta\phi = \delta\phi + k_0\cdot\delta x + n \qquad (6)$$

where all terms are evaluated in parts of a circle. The appropriate integer n, which may depend on frequency, would be chosen in actual practice such that $\Delta\phi$ is a continuous function of frequency and the derived phase velocity is reasonably close to an expected value based on a priori knowledge. (When $\eta = 1$ phase discontinuities of 0.5 rather than 1.0 circles can occur; then n must take a nonintegral value to assure continuity of $\Delta\phi$.) By including this factor we are modeling the actual measurement process. It then follows, e.g., that a negative value of $\delta\phi$ corresponds to a derived wave number less than the desired (fundamental mode) wave number and thus to a positive perturbation of the desired phase velocity. Figure 2 shows a contour plot of $\delta\phi$ in parts of a circle as a function of ϵ and γ for $\eta = 0.25$. Since $\delta\phi$ is periodic in both its ϵ and γ dependence (with period of 1 circle in both variables), this diagram contains the value of $\delta\phi$ for any value of ϵ or γ. Because $\gamma = \delta k\cdot\delta x$, the diagram also contains information for any frequency or station spacing.

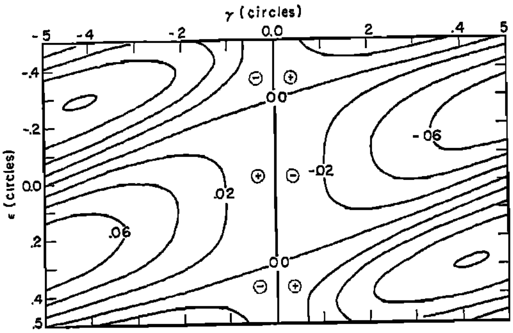

Fig. 2. The dependence of the phase perturbation $\delta\phi$ on the variables ϵ and γ for $\eta = .25$. Contour interval $= 0.02$ circle. The relative location of areas of positive and negative $\delta\phi$ does not depend on η.

Changing the value of η when $\eta < 1$ changes the absolute value of $\delta\phi$ but not the relative regions of positive and negative perturbation. A similar diagram can be constructed when $\eta > 1$, that is, if we consider the higher mode to be perturbed by the fundamental mode. The corresponding diagram for $\eta = 1.0$ is peculiar in that $\delta\phi$ exhibits discontinuities of 0.5 circles. Using the information contained in diagrams such as this, I investigated the phase velocity perturbation for a single earthquake and an ensemble of earthquakes across a given station pair.

Single earthquake. I chose for the structure between the stations A and B the Gutenberg-Birch continental model as given by *Anderson and Toksöz* [1963]. Phase velocities for this model are given in Figure 3. Assuming for illustration that $x_A = 4000$ km, $x = 600$ km, and $\eta = 0.5$, 1.0, and 2.0, and by further assuming that the source phase for the two modes is identical, the phase velocities can be used to calculate γ and ϵ as functions of period. With this information the perturbation of the fundamental mode phase velocity and thus the measured phase velocity was computed. Table 2 contains the pertinent information for $\eta = 0.5$. The measured phase velocities are illustrated in Figure 4. It is interesting that a separation of the interfering waves into perturbing and perturbed terms is justified by the behavior of the measured phase velocity. In each case the perturbation causes an oscillation around the 'normal' curve. If the two interfering waves are of equal amplitude, the resulting phase velocity curve is simply an inverse average of the individual curves. The discussion below assumes that the perturbing term is the higher mode.

Note in Figure 4 that for each wave I have assumed a constant η for all periods. Since, however, changing η does not change the sign of the perturbation, a period-dependent η would give a similar curve. The general character of the resulting measured phase velocity curve is oscillatory rather than consistently biased to high values, as observed by *Aki and Kaminuma* [1963] and *McEvilly* [1964]. It is conceivable, however, that the right combination of structure, epicentral distance, and source depth could give a curve that was apparently biased in one direction. Also note that the negative perturbation in Figure 4 for periods between 50 and 80 seconds is fortuitous and could just as well have been positive. Indeed, if one were able to move the station pair away from the origin (or toward it) and continuously measure phase velocities, the perturbation at any frequency would oscillate between positive and negative values.

Ensemble of earthquakes. Although it might be possible for a given earthquake to give apparently biased phase velocities over a range of periods, the reality of this bias can be determined by considering phase velocities measured from several events at different distances and source depths. Then for a given period γ

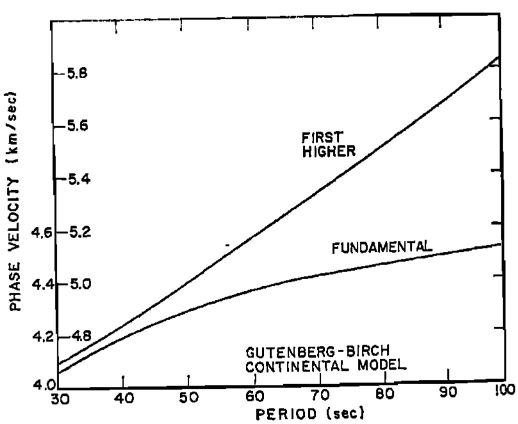

Fig. 3. Phase velocity curves for the fundamental and first higher Love modes for a continental structure [from *Anderson and Toksöz*, 1963]. The vertical scales of the two modes have been displaced relative to one another.

TABLE 2. Illustration of Phase Velocity Perturbation when $\eta = 0.50$, $x_A = 4000$ km, $\delta x = 600$ km, and the Gutenberg-Birch Continental Structure is Assumed

T, sec	$\epsilon = \delta k \cdot x_A$, circles	$\gamma = \delta k \cdot \delta x$, circles	$\delta\phi$, circles	$\phi_0 = k_0 \cdot \delta x$, circles	$dc/c = -\delta\phi/\phi_0$, circles
100	−.02	.299	−.090	1.32	.068
90	.05	.308	−.065	1.47	.044
80	.14	.321	.000	1.68	.000
70	.24	.337	.140	1.94	−.072
60	.38	.356	.155	2.29	−.068
50	−.40	.390	−.070	2.79	.025
40	.18	.477	.140	3.58	−.039
35	−.34	−.451	−.145	4.16	.035

is fixed, and we can consider the perturbation $\delta\phi$ as a function of the phase difference ϵ at station A. For an ensemble of earthquakes we can consider ϵ to be a random variable with a uniform probability distribution. We then ask if the expected value of $\delta\phi$ is nonzero (indicating a bias). The expected value of $\delta\phi$ can be calculated by simply integrating the values in Figure 2 vertically for a given γ. The result of this for all values of γ and a wide range of η was essentially zero. Thus no bias in the phase velocity should occur when measurements are made over a sufficiently random ensemble of earthquakes (sufficiently random meaning that ϵ be a random variable with uniform probability distribution).

Conclusions

By studying the effect of higher mode interference on measured phase velocities, I conclude that the first higher Love mode can be an important contaminant of the fundamental mode, but that interstation phase velocities measured over an ensemble of events should show scatter but no strong bias. It is possible for the measurements of one event to show an apparent bias over a limited period range, but the sign of this perturbation can be positive or negative. In view of this, it would seem that the apparent inconsistency between Love and Rayleigh wave phase velocities cannot be explained simply as an effect of the first higher mode. The results of this study do point up, however, one source of experimental scatter in Love wave phase velocity measurements and argue for using a redundancy of events whenever possible.

Although the investigation was prompted by the possibility of higher mode interference, the results are equally applicable to a model of multipath interference of two similar mode waves in which the 'perturbing' wave is propagating at an angle across the measurement array. In this case the apparent phase velocity of the perturbing wave is higher than normal, and the same analysis used in this paper applies.

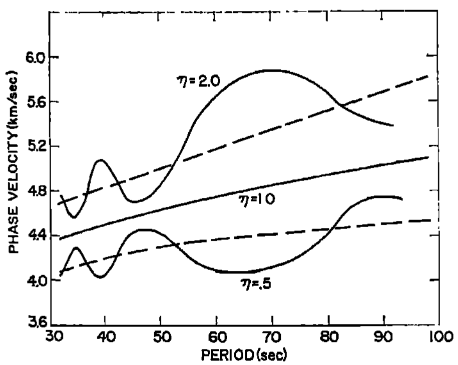

Fig. 4. Measured (perturbed) and unperturbed phase velocities for several values of η. The unperturbed values are indicated by dashed lines.

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