Comparing Shear-Wave Velocity Profiles From Inversion of Surface-Wave Phase Velocities with Downhole Measurements: Systematic Differences Between the CXW Method and Downhole Measurements at Six USC Strong-Motion Sites

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INTRODUCTION

Shear-wave velocity profiles are used in a variety of engineering and seismological applications, and several methods are available to determine the velocities, including borehole and surface-wave methods (Woods and Stokoe, 1985; Stokoe *et al.*, 1988; Stokoe *et al.*, 1994; Poran *et al.*, 1994; Poran *et al.*, 1996). Surface-wave methods offer the advantage of being non-invasive and less costly than boreholes, but their reliability needs to be established by comparison with more direct methods. The best way to do this is a "blind" test in which the velocities determined independently from surface-wave and borehole measurements at the same site are compared. By "blind" we mean that the determination of the shear-wave velocity profile by the two methods were done independently of one another. It is surprisingly difficult to find situations in which the velocity determinations are truly independent of one another. We report on one such case in this paper. In addition, we introduce a method of comparing two velocity profiles that we think has advantages over the usual visual comparison of plots of the velocities versus depth.

SHEAR-WAVE VELOCITY DATA USED

In this paper we focus on a group of shear-wave velocity determinations made at the University of Southern California (USC) accelerometer sites. In conjunction with USC, Vibration Instruments Company (VIC), Ltd. from Japan carried out a field investigation in 1993 using a non-invasive Rayleigh-wave measurement method, which they call the CXW method. For a complete description of the investigation and technique used, see the VIC final report (1993). Shear-wave velocity profiles were determined at 128 sites. The shear-wave velocities at these sites are of particular interest because of the important ground-motion records obtained at these sites from numerous earthquakes, including Whittier Narrows 1987, Landers 1992, and Northridge 1994. In addition, the VIC measurements comprise a large fraction of the southern California boreholes included in the Pacific Engineering & Analysis and SCEC borehole databases (Wills, 1998).

Subsequent to the publication of the CXW results, borehole velocity profiling was done at three of the USC sites (Gibbs *et al.*, 1996); the borehole velocity determinations did not make use of the CXW results. We searched for other USC sites with borehole velocity profiles located within 2 km and for which it is unlikely that the geology changes significantly between the borehole and the USC sites, according to John Tinsley of the U.S.G.S. (personal communication, 1997). We found three such sites, for which the borehole velocities were reported by Gibbs *et al.* (1980, 1996). Even though the velocities at two of the sites were published in 1980, before the CXW results, the CXW interpretations made no use of the borehole results. The six sites and the distances between them are listed in Table 1.

The shear-wave velocities of the Gibbs *et al.* (1980) profiles, at SHS and KAT, were interval velocities, derived with no effort made to fit the travel times to a set of layers. To insure consistency at all sites, we fit layered velocity models for SHS and KAT to the shear-wave travel times tabulated in Gibbs *et al.* (1980), using the same procedure as in Gibbs *et al.* (1996). Layered velocity models for the other borehole sites were taken from Gibbs *et al.* (1996). The CXW layered model shear-wave velocities are those tabulated on one of the figures for each site in the VIC final report (1993).

A METHOD FOR COMPARING VELOCITY PROFILES

The traditional method of comparing several estimates of shear-wave velocities (V) is to make a visual comparison of V plotted as a function of depth. This is shown in Figure 1a.

TABLE 1.
USC sites with nearby USGS boreholes used in this comparison. Distance between sites was calculated from latitude and longitude coordinates for the sites published in Gibbs *et al.* (1980, 1996) and Anderson *et al.* (1981).

USC Site	USC Site Name	BH Site	Borehole Site Name	Intersite Dist. (km)
USC01	Light and Life School	SHS	Sylmar High School	0.403
USC03	White Oak Church	WOC	White Oak Church	0.078
USC10	L.A. Fire Station 78	SOW	Sherman Oaks Woodman	1.914
USC53	Epiphany Lutheran Church	ELC	Epiphany Lutheran Church	0.050
USC55	Knolls Elementary School	KES	Knolls Elementary School	0.045
USC90	Cerro Villa School	KAT	Katella School	0.776

The problems with this method are that it is subjective and qualitative. In addition, it does not emphasize the relative differences in travel time between the two estimates. Travel time is a more fundamental parameter in site response than is velocity. In the traditional comparison, absolute differences in V at great depths stand out, but low velocity layers near the surface that have a greater effect on travel time go unnoticed. This will be clearly shown in this paper.

Rather than a visual comparison of two different estimates of shear-wave velocity, we use a quantitative comparison that is related to the relative site amplification produced by the two profiles. We first convert velocity to travel time (tt) as a function of depth (z)

$$tt(z) = \sum\left(\frac{h_i}{V_i}\right), \tag{1}$$

where h_i is the layer thickness and V_i the interval velocity (in what follows, all references to velocity should be taken to mean shear-wave velocity). From the travel time we compute average $\overline{V}$ from the surface to a given depth using the equation

$$\overline{V(z)} = \frac{z}{tt(z)} \tag{2}$$

An example is shown in Figure 1b. We then use the quarter-wavelength amplification approximation (Joyner *et al.*, 1981) to form a ratio of amplifications produced by the different velocity models. The quarter-wavelength amplification approximation for a particular velocity profile is given by

$$A(f(z)) = \sqrt{\frac{\rho_r V_r \cos(i_r)}{\overline{\rho}\,\overline{V}\,\overline{\cos(i)}}} \tag{3}$$

where $A(f)$ is the motion at the surface divided by the motion at the surface of a halfspace with material properties equal to those at the reference depth (for example, the source depth). ρ_r, V_r, and i_r are the density, shear-wave velocity, and angle-of-incidence at the reference depth, and the equivalent quantities averaged from the surface to a depth z are given by $\overline{\rho}$, $\overline{V}$, and $\overline{\cos(i)}$. It is probably more correct to use an average of the product rather than the product of the averages in equation (3). The relatively small vertical variations in ρ and cos i, however, should lead to similar computational results in either case. Using Snell's law, the quantity $\overline{\cos(i)}$ is approximated by

$$\overline{\cos(i)} = \sqrt{1 - \left[\left(\frac{\overline{V}}{V_r}\right)\sin(i_r)\right]^2}\ . \tag{4}$$

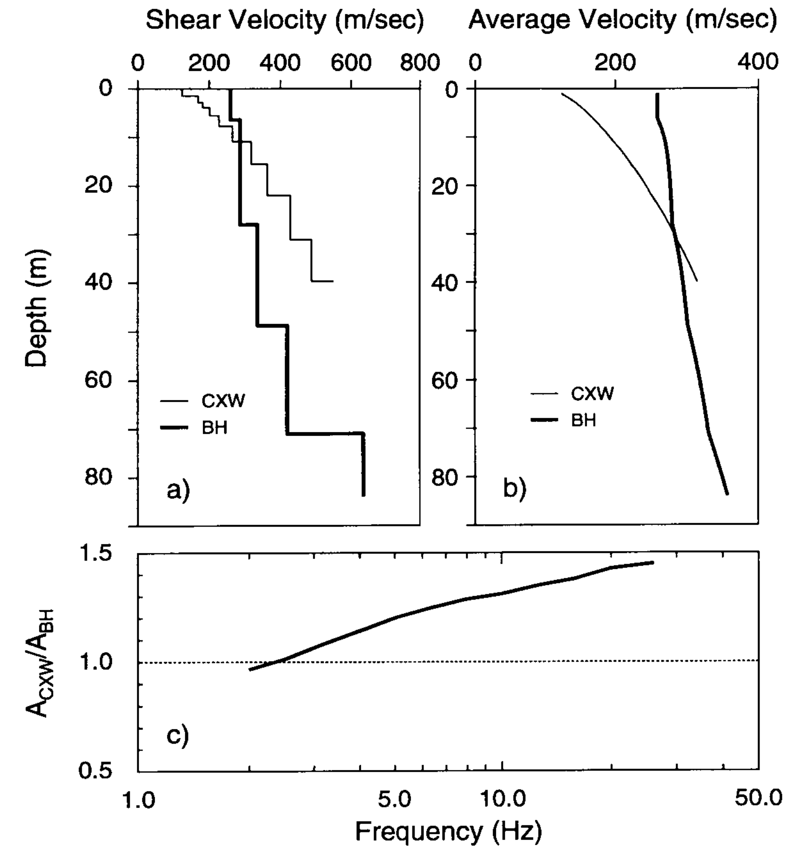
The frequency corresponding to this amplification is given by

$$f(z) = \frac{1}{4tt(z)} \tag{5}$$

With this definition and using equation (2), it is easy to show that the depth z is equal to a quarter wavelength ($z = 0.25\overline{V}/f$).

To compare the amplification for two velocity profiles, we assume that the velocity models (in this case the borehole (BH) and CXW velocities) converge to the same bedrock velocity at the reference depth, that the densities are the same for both models, and that the angle-of-incidence (i_r) equals 0.0 (only the first of these assumptions has much influence on the results; the other two assumptions are imposed to minimize the number of parameters that need to be specified). For each model we then find the depths z corresponding to a specified frequency f, compute the amplifications, and divide these amplifications. This procedure yields the following simple equation for the relative amplification from the BH and CXW velocity models:

$$\frac{A_{CXW}}{A_{BH}} = \sqrt{\frac{\overline{V_{BH}}}{\overline{V_{CXW}}}} \tag{6}$$



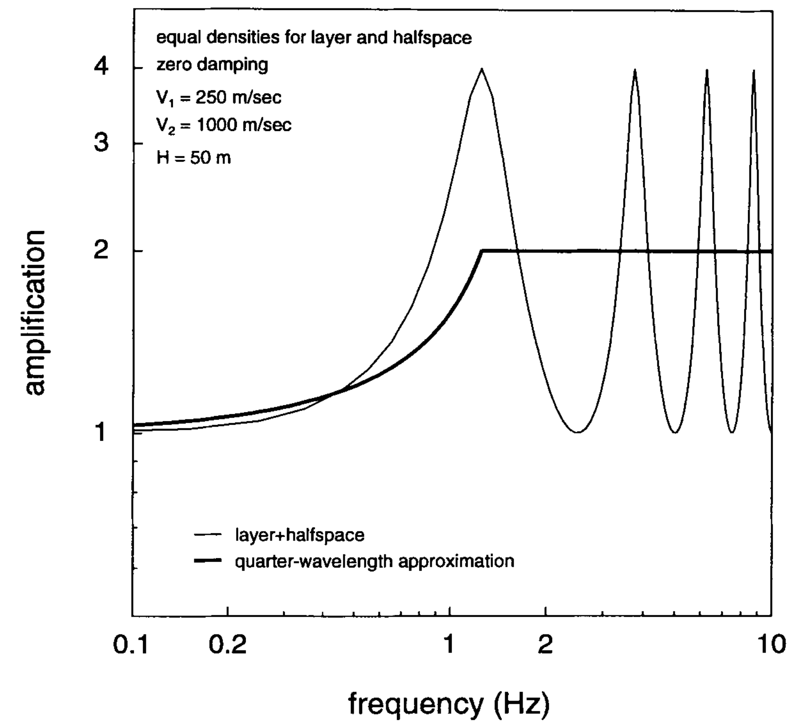
▲ **Figure 1.** Demonstration of analysis method, using shear-wave velocities at Epiphany Lutheran Church (USC53) as an example: (a) shear-wave velocity profiles from CXW and borehole (BH) methods, (b) time-weighted average shear-wave velocity, (c) amplification ratio vs. frequency.

where it should be remembered that the depths over which the averages are computed will in general be different for each model (such that the quarter-wavelength frequency is the same). An example of this calculation is shown in Figure 1c.

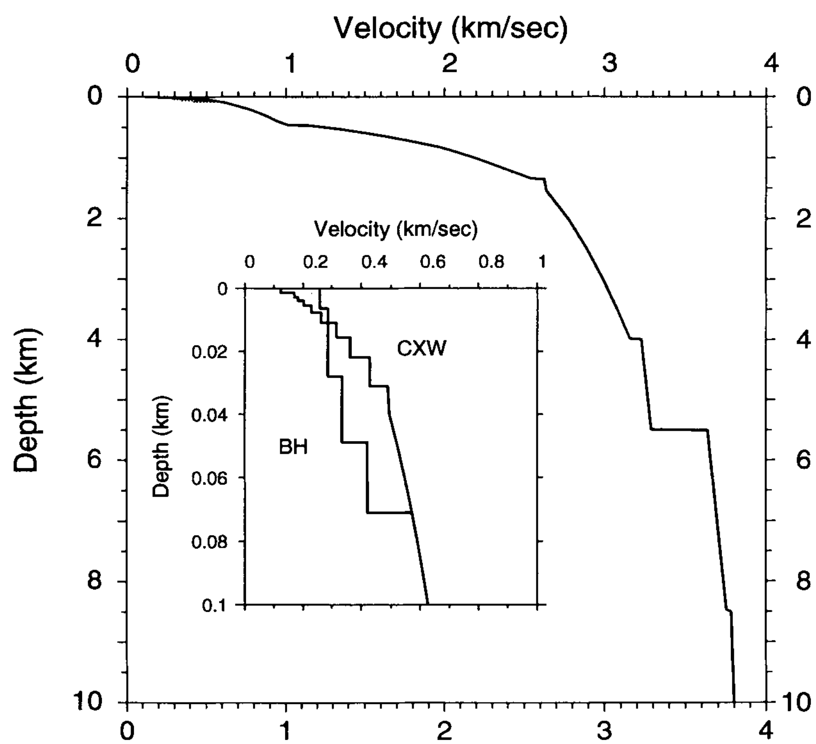
This quantitative method for comparing velocity profiles is simple and reproducible, and it relates the differences in the velocities to a useful quantity—the site amplification. The method is not sensitive to assumptions about the velocity structure between the bottom of a profile and the reference depth (the amplification is only computed for frequencies higher than that corresponding to the bottom of the profile). This method does not account exactly for resonance due to discontinuities in seismic velocity, but it gives an amplification function that is generally comparable to a smoothed version of the exact theoretical amplifications (Boore and Joyner, 1991, 1997; Day, 1996). For example, for vertically propagating *SH* waves in a layer over a half-space, the peak amplification is $\rho_2 V_2 / \rho_1 V_1$, where the subscripts "1" and "2" refer to the layer and halfspace, respectively. It is easy to show that the rms of the amplification function is given by

$$\sqrt{\rho_2 V_2 / \rho_1 V_1},$$

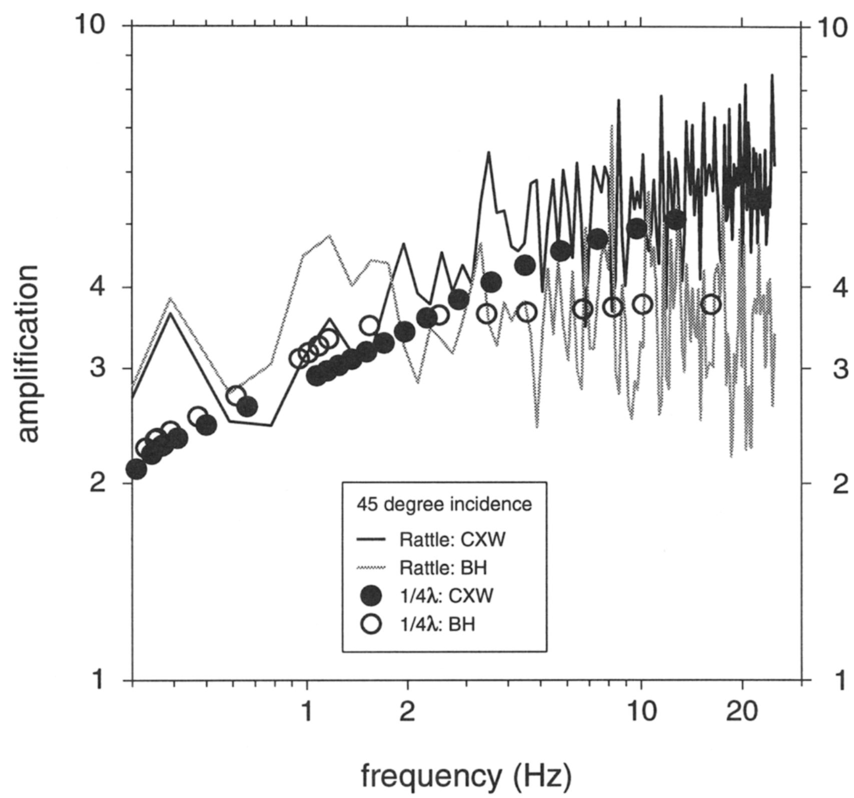
which is the amplification that would be computed by the quarter-wavelength approximation. We show a specific example of this in Figure 2. To investigate the nature of the approximation for more complicated models, we consider the USC53 site used in the example in Figure 1. We wish to compare the ratio of amplifications given by the approximate method and by wave propagation that accounts for resonance effects. For the wave propagation we use the Haskell matrix method, implemented by C. Mueller's program RATTLE. In order to use this, however, we must specify the velocity structure to depths greater than the depths to the bottom of the profiles. For the sample site, this velocity was provided by Harold Magistrale (written communication, 1998), using an updated version of the Magistrale *et al.* (1997) model for the 3-dimensional velocity structure in the Los Angeles region. We blended the BH and CXW velocity profiles with Magistrale's velocity profile (Figure 3). The amplifications computed both from RATTLE and from the quarter-wavelength approximation are shown in Figure 4, from which it can be seen that the relative amplification for the BH and CXW velocity profiles is adequately given by the quarter-wavelength approximation (it should also be noted that the RATTLE results would have to be heavily smoothed if ratios of the response were to be computed; this is not necessary for the quarter-wavelength approximation).



▲ **Figure 2.** Amplification for a layer over a halfspace from the complete wave solution (thin line) and from the quarter-wavelength approximation to the amplification (thick line). The model parameters are given in the figure. Note that the wave propagation gives spectral peaks equal to the ratio of the seismic impedances (4.0 in this case), while the quarter-wavelength approximation to the amplification attains a high-frequency level equal to the square root of the ratio of the seismic impedances (2.0). This square root of the impedance ratio also equals the rms of the spectral amplification taken over whole cycles of the amplification in the frequency domain.



▲ **Figure 3.** Velocities for USC53 and ELC used in comparing the full wave and quarter-wavelength computations of amplifications. The lower parts of the USC53 and ELC velocity profiles were joined onto a shear-wave velocity profile provided by H. Magistrale for this particular site.



▲ **Figure 4.** Amplifications using wave propagation (RATTLE) and the quarter-wavelength approximation for the CXW and borehole (BH) velocity profiles, extended to depth as in Figure 3. For this run, an SH plane wave was assumed to be incident at 45 degrees at a depth for which the shear-wave velocity was 3.7 km/sec and the density was 2.8 gm/cm^3. The density was assumed to increase linearly with velocity from a value of 2.0 gm/cm^3 at a velocity of 0.5 km/sec to 2.8 gm/cm^3 at a velocity of 3.5 km/sec, with constant density of 2.0 and 2.8 gm/cm^3 for lower and higher velocities, respectively.

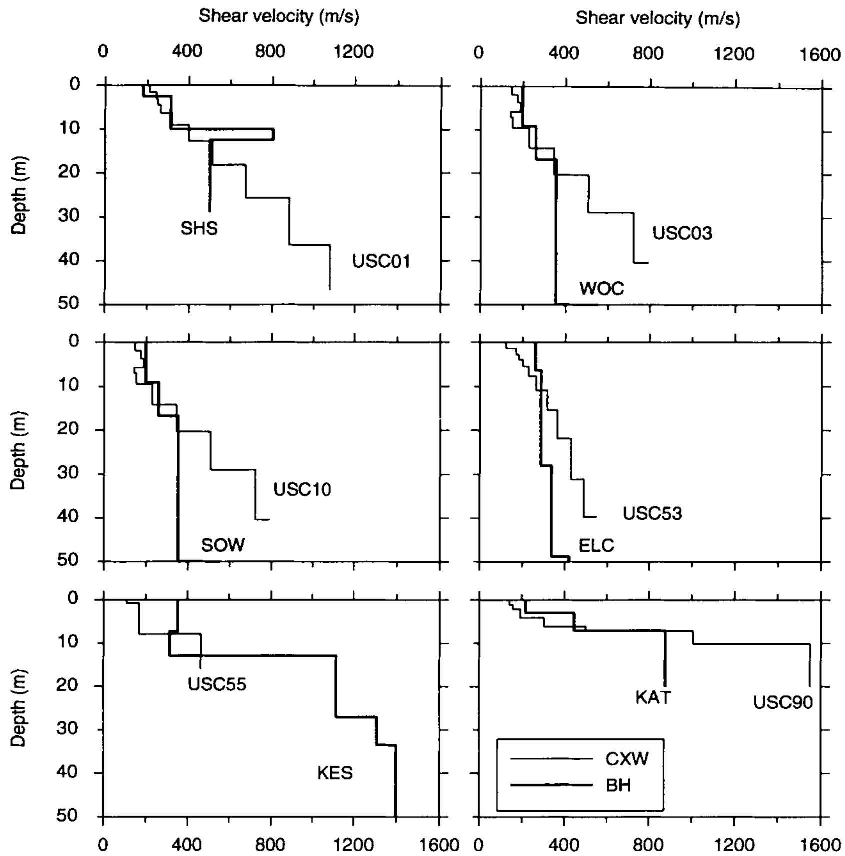
RESULTS

Amplification ratios were determined for a frequency range of 2–35 Hz, depending on the site. The highest frequency was determined by the travel time to 2.5 m depth (maximum resolution) in the borehole data. The lowest frequency was determined by the minimum of the travel times to the bottom of the two velocity profiles.

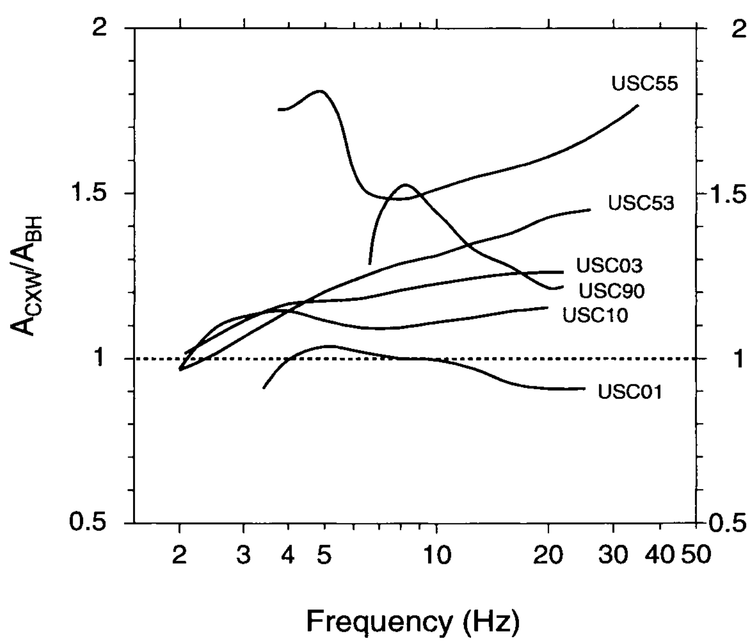
The *V*(*z*) plots are shown in Figure 5. In the plots the most obvious difference between the borehole and CXW velocities are the high and increasingly divergent CXW velocities at depth. Less noticeable, however, are the consistently low CXW velocities at shallow depths. Based on these figures, one might think that the CXW amplifications would be low compared to those from the borehole velocities. This is not the case, as shown by the amplification ratios computed for the six USC sites, shown in Figure 6. The most obvious feature is the systematic overprediction of amplification from the CXW velocities. The overprediction increases at higher frequencies but is always less than a factor of two. The reason for this is that the lower shear-wave velocities in the CXW model at shallow depths affect the travel time much more than do the velocities of the deeper layers. Nonetheless, if the higher velocity trends in the CXW results persist to still greater depths, the amplification ratio would be less than unity for lower frequencies (<2 Hz).

It is interesting that the largest amplification ratio is for the most closely colocated sites (USC55 and KES, 0.05 km apart), while the most widely separated stations (USC10 and SOW, 1.9 km apart) have an amplification ratio much closer to unity. If the opposite were the case, we would question our pairing of noncolocated sites.

Under NEHRP provisions (BSSC, 1994), sites are categorized using $\overline{V}_{30}$. We evaluated $\overline{V}_{30}$ for all of the sites and assigned NEHRP site classes accordingly (Table 2). Two sites, USC55 and USC90, did not have deep enough CXW results to be classified, but the site classes can be estimated by requiring reasonable values of the extrapolated velocities. At borehole station KAT (and the nearby CXW site USC90), the velocities are given to only about 20 m. The average velocities to 20 m from the borehole and CXW values are 532 and 471 m/sec, respectively. Assuming that no large velocity reversals occur between 20 and 30 m, this means that the site must fall into a higher-velocity class than D. In order to fall into class B, the shear-wave velocity between 20 and 30 m would have to be in excess of 4100 m/sec; this is unreasonable, and therefore we can be certain that the site should be placed in the class C category. A similar argument



▲ **Figure 5.** Shear-wave velocity profiles obtained from the CXW method at six USC sites (thin lines), compared with velocities from downhole velocity profiles in boreholes drilled at or near the USC sites (thick lines). See Table 1 for key to abbreviations.



▲ **Figure 6.** Amplification ratios vs. frequency at the six USC sites in our study. Notice that the CXW results give consistently higher ground motions at frequencies above 2 Hz.

TABLE 2.
Comparison of average shear-wave velocity from the surface to a depth of 30 m ($\bar{V}_{30}$) from the borehole and CXW results, and assignment of NEHRP site classes on the basis of $\bar{V}_{30}$.

				NEHRP Site Class	
USC Site	**V30(m/sec)**	**BH Site**	**V30(m/sec)**	**CXW**	**BH**
USC01	423	SHS	394	C	C
USC03	268	WOC	279	D	D
USC10	258	SOW	264	D	D
USC53	282	ELC	283	D	D
USC55	NA	KES	564	NA[a]	C
USC90	NA	KAT	NA	C[b]	C[b]

a. Would be class C if average velocity between 16 and 30 m is ≥ 863 m/sec, which seems likely in view of the BH velocity profile (Fig. 5).

b. Site class based on average over 20 m; a velocity in excess of 4100 m/sec between 20 and 30 m is required to put the site into class B.

cannot be made for USC55; from the CXW results, the site could be placed in classes C or D. The site would fall into class C if the average velocity between 16 and 30 m is ≥863 m/sec, which seems likely in view of the BH velocity profile (Fig. 5).

Despite the low velocity layers near the surface in the CXW results, average V at a depth of 30 m ($\bar{V}_{30}$) was very close to the borehole model for the six sites we evaluated (Table 2), and therefore no differences exist in the NEHRP site classifications based on the CXW and the BH results. It is not clear why the average V from the CXW results is so close at 30 m to that obtained from the borehole results, but since the shear-wave velocities from the CXW results are usually lower than those of the borehole model at shallow depths and higher at greater depth, the differences balance out somewhere in between (Figures 5 and 6). This finding is in contrast to Wills' finding that $\bar{V}_{30}$ determined from the CXW results for all 128 USC sites is systematically high compared to $\bar{V}_{30}$ from boreholes (Wills, 1998). (As noted above, few boreholes were colocated at the USC sites, so Wills' results are based on a statistical comparison of average velocities for measurements made in presumably similar geologic materials).

CONCLUSION AND DISCUSSION

At six colocated or closely located sites we found that the shallow velocities from the CXW method are low, and at deeper depths, the CXW velocities are generally larger than the borehole velocities, with the difference increasing with depth, in agreement with the inference of Wills (1998) from comparisons of average velocities from CXW and borehole profiles in generally comparable soils. Using the quarter-wavelength approximation to site amplification as a new way of comparing two shear-wave velocity profiles, we find that the lower CXW velocities are more important for the site amplification than are the higher velocities at greater depths, at least for the limited depth range for which shear-wave velocities from the CXW measurements have been tabulated. The lower shallow CXW velocities lead to increased amplification relative to the amplifications from the borehole velocities.

What are the implications of the differences in the CXW and the borehole velocities? While it is true that the shear-wave velocities determined from surface-wave measurements represent a spatial average over the dimensions of the instrumental array, and therefore the velocities from the two methods will not necessarily be comparable if significant lateral variations in velocity are present, the spacing between the instruments used in the surface-wave results discussed in this paper are small enough that such differences are expected to be small. Being a more direct measure of the subsurface velocities, we assume that the borehole velocities are correct and that the systematic differences we and Wills (1998) find indicate that the CXW velocities contain systematic errors. In view of this, we recommend caution in using the VIC results as published in 1993 and incorporated into some recent databases.

This is not to say the results of the CXW method as presently used are not to be trusted. According to C. Poran (written commun., 1997), significant improvements have been made in the CXW method, both in data gathering and in inverting the phase velocities for shear-wave velocity with depth. Furthermore, alerted by our and Wills' results, C. Poran is in the process of reevaluating the shear-wave velocity inversions at all of the USC sites (C. Poran, written commun., 1998). ■

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