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LETTERS TO THE EDITOR

EFFECT OF THE FREE SURFACE ON CALCULATED STRESS DROPS

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INTRODUCTION

In converting moment (or average displacement $\bar{D}$) and fault dimensions to stress drop, $\Delta\sigma$, an equation of the form

$$\Delta\sigma = C\mu(\bar{D}/l) \tag{1}$$

is usually used, where μ is the rigidity of the medium and l is a measure of fault dimension. C, a numerical factor related to the shape of the fault, is determined by the solution to the crack problem in which constant stress is applied over the faces of the fault. For faults imbedded in an infinite medium

$$C = 7\pi/16 \tag{2}$$

$$l = R \tag{3}$$

for a circular fault of radius R (Keylis-Borok, 1959) and

$$C = 4/\pi \tag{4}$$

$$l = W \tag{5}$$

for a two-dimensional fault of width W and dislocation along the long axis (Knopoff, 1958). Due to the complicating effect of the free surface, the shape factor C for a fault near the free surface is readily available only when the two-dimensional fault just discussed breaks to the surface. In this case the l factor given by equation (5) applies if

$$C = 2/\pi \tag{6}$$

(Chinnery, 1969; Kanamori and Anderson, 1975). Alternatively, the shape factor could be given by equation (6), with the l factor equal to the half-width for buried faults and the total width for faults which break to the surface. In either case, the inferred stress drop on a long fault with given width W and average displacement $\bar{D}$ will differ by a factor of 2 depending on whether the fault breaks the surface or is deeply buried (assuming the same rigidity). The question arises as to which shape factor (or l factor) should be used for shallow faults which do not break to the surface. This is an important question, for a number of shallow earthquakes seem to have larger displacements at depth than would be inferred from surface observations (e.g., 1952 Kern County, Hanks *et al.*, 1975; 1966 Parkfield, Scholz *et al.*, 1969).

RESULTS

To answer this question for rectangular, vertical strike-slip faults, we solved for the fault displacement, and thus the average slip, consistent with a constant stress drop for various depths h to the upper edge of the fault from the free surface. In keeping with Knopoff's model we define $l = W$ (fault width) and determine C from equation (1), as a function of $\Delta\sigma$, μ, W, L, $\bar{D}$, where L = fault length. A Poisson solid is assumed. The computations leading to the average slip are based on a discretization of the fault area into a number of segments and the use of a Green's function giving the stress at a point due to a rectangular fault with constant displacement buried in a half space. The constant slip on each fault segment is determined such that the stress in the center of each segment, due to a superposition of stresses from all of the segments, is constant. Details can be found in Dunbar (1977).

The results are shown in Figure 1 for a range of length-to-width ratios and burial depths (normalized by fault width). The influence of the free surface diminishes rapidly as the depth of burial increases. For depths greater than 0.1 of the fault width the C

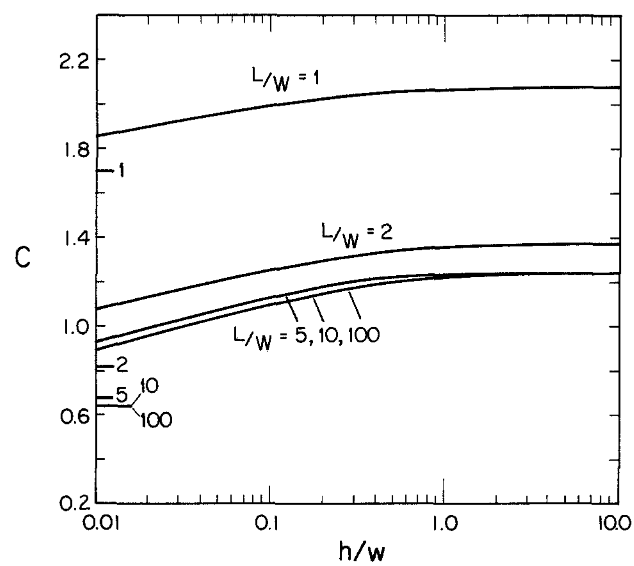

FIG. 1. Shape factors for a vertical strike-slip fault as a function of normalized depth of burial for various L/W ratios, assuming a constant stress drop over the fault surface. The $h/W = 0$ values are indicated by tick marks.

factor is within 16 per cent of the infinite media result. This is consistent with the results of Oda and Hirasawa (1976) who solved a similar problem for $L/W = \infty$. In many cases, then, stress drops on shallow faults can be estimated from the infinite media results. We also see that for practical purposes faults with L/W greater than about 5 can be treated as two-dimensional (infinitely long) faults.

Discussion

Similar computations can be made for dip-slip faults of arbitrary orientation, but as Chinnery (1969) noted, the symmetry of the stress tensor can lead to inconsistencies between the zero stress free surface and the finite stress acting on the fault surface, especially if the fault intersects the earth's surface. In other words, a dip-slip fault with constant stress across its surface may be unphysical. It should be noted, however, that although these inconsistencies do not exist in the case of an infinitely long strike-slip fault, they may enter to some extent in our calculations for finite faults, especially near the fault ends. Furthermore, although the sensitivity of the results to the burial depth is mathematically correct for the problem as stated, large stresses will exist in the region between the surface and the fault, and we would expect the fault to break to the surface. The observational fact that this does not always happen for shallow faults may be related to the decrease in rigidity usually found near the earth's surface. This may mean that the transition between the free surface and infinite media C factors will be less rapid than shown in Figure 1. Finally, as Madariaga (1977) pointed out, the stress drop determined from equation (1) may not correspond to the real average stress drop across faults in which the stress drops are not constant over the surface. The variable stress changes can lead to large differences in the shape factors. For example, Table 1 shows the shape factor relating the peak stress drop to the average slip in models in which the slip is constant (Chinnery, 1969) and in which the slip $D(x, y)$ goes as

TABLE 1

VALUES OF C FOR A STRIKESLIP FAULT IN AN INFINITE MEDIUM, WHERE $\Delta\sigma$ OF EQUATION (1) IS THE MAXIMUM STRESS DROP ON THE FAULT SURFACE

Author	Length/Width			
	1	2	5	∞
Chinnery (1969)	1.1	0.8	0.6	0.6
Sato (1972)	3.4	2.4	1.9	1.9

C is defined in equation (1) with l equal to the total fault width. $\bar{D}$ is the average slip on the fault.

TABLE 2

VALUES OF C FOR A STRIKESLIP FAULT IN AN INFINITE MEDIUM WHERE $\Delta\sigma$ OF EQUATION (1) IS THE AVERAGE STRESS CHANGE ON THE SLIP SURFACE

Author	Length/Width			
	1	2	5	∞
Sato (1972)	0.4	0.3	0.2	0.4
This paper	2.1	1.4	1.3	1.3
Knopoff (1958)	—	—	—	1.3
Keylis-Borok* (1959)	2.4	—	—	—

* Circular fault, converted to an equivalent square fault using equations (1), (2), and (3) with $R = W/\sqrt{\pi}$.

$$D(x, y) = k\bar{D}[(W/2)^2 - x^2]^{5/2}[(L/2)^2 - y^2]^{5/2}$$

where k is a normalization factor and x, y are distances from the center of the fault (Sato, 1972). Table 2 shows the shape factors when average stress change over the fault surface rather than peak stress drop is used in equation (1) to define C. The Knopoff and Keylis-Borok models assumed constant stress drop and thus are close to our results for the appropriate L/W ratio. As is clear from the tables, a wide range of stress estimates can be obtained from a given size fault and mean dislocation. Most published stress drops seem to rely on the shape factors given by the constant stress drop models.

In spite of the qualifications above, stress drops determined from equation (1) continue to be made, and in view of this we feel it important to define the appropriate shape factor in cases where shallow faults may not reach the surfac :. The stress drop so determined may not correspond to the actual average change in stress across the fault being studied, but it will be consistent with most of the stress drops reported in the literature.

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