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A NOTE ON THE EFFECT OF SIMPLE TOPOGRAPHY ON SEISMIC *SH* WAVES*

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ABSTRACT

Finite difference calculations for *SH*-motion, as well as field observations, show that topography can have significant effects on seismic waves when the incident wavelengths are comparable to the size of the topographic features and the topographic slopes are relatively steep. These effects are frequency-dependent and can range from amplification to deamplification at a given site. For the models considered in this paper, amplifications of 75 per cent as compared to motions produced in regions of no topography were found. A qualitative discussion of the strong-motion accelerogram recorded at Pacoima Dam from the San Fernando earthquake of February 9, 1971, suggests that the recorded accelerations could, at any period, be expected to differ by 25 to 50 per cent from those that would have been recorded if no topographic features were present. The complex terrain precludes a prediction of the actual magnitude or sign of the difference. The present results are probably most significant from the viewpoint of engineering seismology, for the conditions required for a significant topographic effect are most likely to occur in the period range of engineering interest.

Introduction

Much study has been given the amplification of seismic energy due to velocity variations in the Earth, but relatively little attention has been paid to the effect of the Earth's surface topography on incident waves. When seismic wavelengths are much larger than topographic irregularities and slopes of the irregularities are small, the surface topography can be neglected. Although these conditions are met in many seismological investigations, topography may sometimes have a considerable effect on the seismic waves. It is probably in engineering seismology that this effect is most important, for then one may be concerned both with steep slopes and relatively high frequencies and, thus, short wavelengths. This study was prompted by observations, during the recent San Fernando earthquake of February 9, 1971, of topographically correlated high accelerations (Nason, 1971) and the question of the influence on accelerations of the pronounced topographic relief in the vicinity of the Pacoima Dam strong-motion accelerograph, which recorded high-frequency accelerations up to 1.25 g (Trifunac and Hudson, 1971).

Method

Previous attempts to study the effect of topography on seismic waves have been concerned with the scattering of incident waves and conversion of body to surface-wave energy (Gilbert and Knopoff, 1960; Hudson, 1967; Hudson and Knopoff, 1967; McIvor, 1969). The theories used in these studies are not valid for motions in the vicinity of the scatterer, when the slopes are steep, or when the topography and seismic disturbances are of comparable wavelength. In such cases a numerical treatment of the problem is required.

The finite difference method was used in this paper to simulate the propagation of a transient *SH*-disturbance incident on a nonplanar free surface. The material through which the wave propagates was assumed to be perfectly elastic, isotropic, and homogeneous. At a set of grid points, the displacements resulting from the time-marching process were saved, resulting in a set of "seismograms" at sites along the surface. These

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were then subjected to Fourier analysis, and spectral ratios were formed by dividing by the spectra of the wave motion resulting from the transient disturbance incident on a planar free surface. Details of the method can be found in Boore (1970), Boore *et al.* (1971), and Boore (1972). The only new feature in the present computations was the treatment of the free-surface boundary condition. This is discussed in the appendix.

Results

It is unlikely that detailed predictions of the effect of arbitrary topographic features can be made on the basis of computations for a few "characteristic" models. Computations for a whole suite of models may also not yield general conclusions and, furthermore, are beyond the scope of this paper. Thus, for this report, only three, closely-related structures were used (Figure 1). Although the results may not be general,

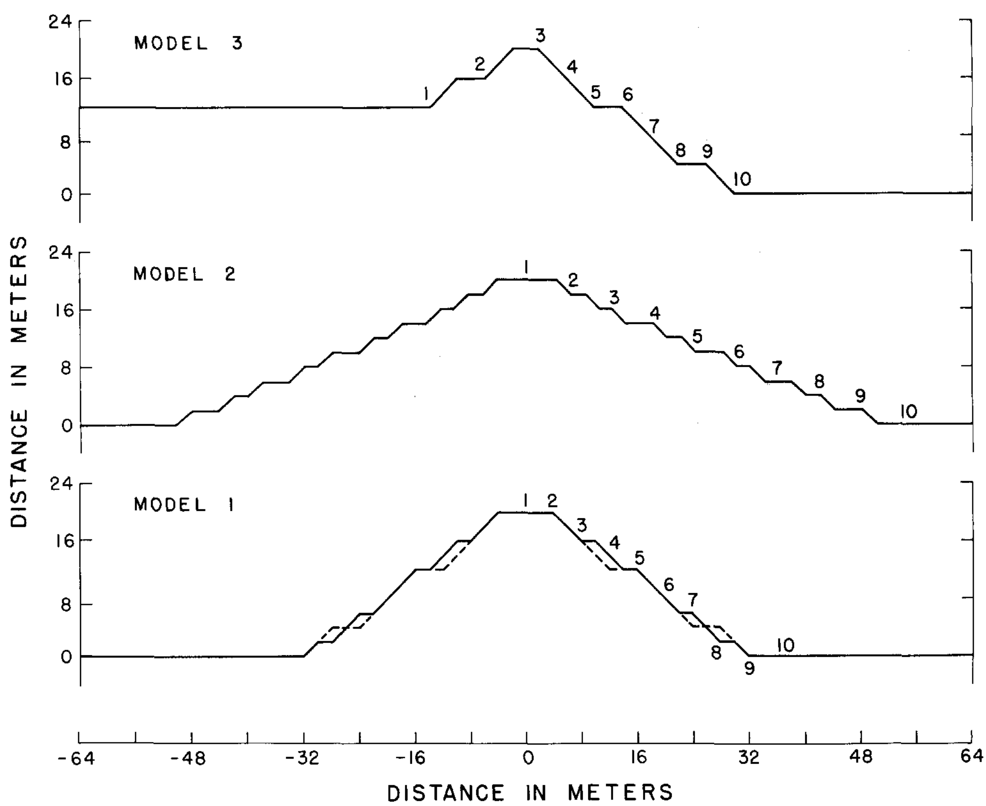

Fig. 1. Three topographic models used in the calculations. Numbers indicate spatial locations at which spectral ratios were formed. *Dashed lines* in model 1 indicate a configuration used in a run with grid spacing twice that of the *solid line* run.

they do show what possible effects topography can have and give some idea of the structure size per wavelength regime in which topography may be important. Two of the models are symmetrical, but have different slopes (35° and 23°), and the other, antisymmetrical model, is an approximation to that described by Nason (1971) upon which he observed shattered earth after the San Fernando earthquake. The models as shown in Figure 1 are made up of flat segments and 45° steps. This approximation to a smooth surface was dictated by the manner in which the surface boundary condition was treated.

The accuracy of the results was tested in several ways. As shown in the diagram for model 1, several calculations were run at grid sizes differing by a factor of two. The answers were nearly identical, indicating both that the solutions are good approximations to the true solution, and that, for the relatively long wavelengths involved (at least 12 grid points per wavelength), the step-like surface looks like a smooth surface to the wave. The presence of contamination from the artificial boundaries, imposed by limits of computer storage, was tested by repeating some calculations with varying distances to the artificial boundaries. The above checks show that the calculations should be accurate to within at least 10 per cent and often are better than 5 per cent. Finally, the results for models 1 and 2 were checked at one frequency with those obtained from the Aki-Larner (AL) method (Aki and Larner, 1970). The comparison for model 1 is shown in Figure 2. The results have the same patterns of amplification and deamplification but

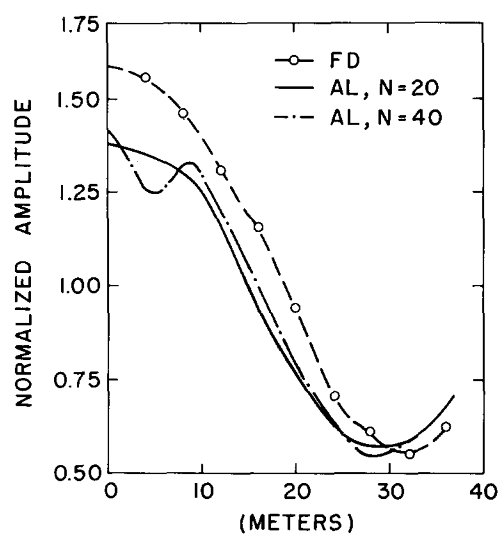

FIG. 2. Comparison of finite difference (FD) and Aki-Larner (AL) results for model 1 at a frequency of 5 Hz (assuming shear velocity of 500 m/sec). *N* is the number of scattering orders used in the AL method. The abscissa is horizontal distance from the center of the hill.

differ by about 15 per cent. That the comparison is not better is not surprising, however, for the AL method is not very accurate for the steep slopes encountered in this problem. As evidence of this, the AL results for 20 scattering orders are probably, as shown in the figure, better than for 40 scattering orders. This is due to the semi-convergent nature of the AL solution. There are other possible reasons for the disagreement between the AL and finite difference solutions: the horizontal periodicity implied in the AL method and the use of steady-state solution can combine to produce contamination of the solution by scattering from neighboring regions of topographic relief, and the models were not exactly the same (the AL solution uses sections of cosine curves to define the surface). The disagreement between the two methods is relatively unimportant to the conclusions of this paper.

In order to extend the range of frequencies, and also to make certain that the results are not dependent on the type of input transient motion used in the numerical solution, the computations in model 1 were made with three different input motions: two Ricker wavelets with overlapping amplitude spectra and an approximation to an impulse. The resulting seismograms for the impulse and for one of the Ricker wavelets are shown in Figure 3 for sites defined in Figure 1. Some of the oscillations in the spike seismogram are dispersion and aliasing effects due to the discrete grid (Boore, 1972). The spike results clearly show that the motion is composed of at least three phases: the direct wave, a wave reflected off the opposite side of the topographic bump, and diffracted energy, probably from the corner at the bottom of the slope. The motion following the main spike at site 1 is interpreted to be diffracted energy; its main lobe arrives at a time equivalent to a slant path from the bottom corner to the top, and no geometrical rays should arrive in that time range. The diffracted and reflected energy are important in determining the character of the wave form in the frequency range of the Ricker wavelets. This explains why, in Figure 3, the main spike arrival shows only a small amplification at site 1 as compared to the reference trace, but the other seismograms, which can be thought of as a Ricker wavelet convolved by the spike seismograms, show much larger amplifications.

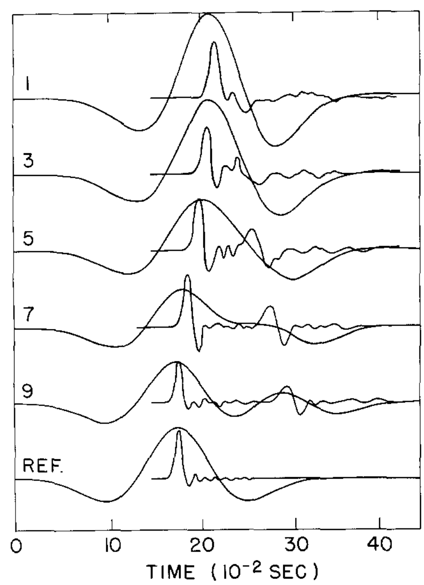

FIG. 3. Computed seismograms at selected sites in model 1. Reference trace is surface motion for a model without topographic relief. Time scale is based on assumed shear velocity of 500 m/sec.

The amplitude spectra of the input and the spectral ratio versus frequency plot from sites 1 and 10, computed from the Fourier transforms of the seismograms, are shown for each input motion in Figure 4. The results from the three computations agree to at least 5 per cent in the region of overlap, except at the lowest frequency plotted. There the results from the longer-period inputs B and C tend to approach unity as frequency decreases. This is, of course, just what we expect and is the basis for neglecting surface topography for relatively long-period seismic waves.

The spectral ratio results for models 1, 2, and 3 are given in Figures 5, 6, and 7, respectively. Before Fourier analysis, the times series were multiplied by an exponential time window with a decay time equivalent to dampings of 20 per cent to 2 per cent depending on frequency. This time window was used to facilitate comparison with the AL results, but numerical experiments showed that the results were not materially affected by the window used: decreasing the decay time by a factor of two resulted in an increase of less than 2 per cent in the amplitude ratio. The reason for this slight dependence in the time window is the absence of any layering in the model. Thus, multiple reverberations, which are relatively more affected by damping than are first motions, are excluded. A further discussion of the use of a time window is contained in Boore *et al.* (1971).

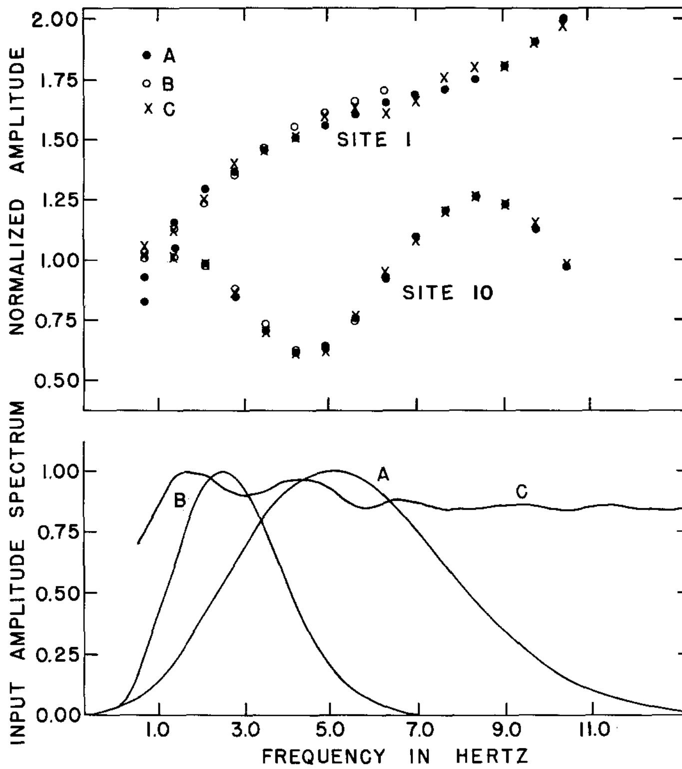

FIG. 4. (*lower*) Amplitude spectrum for three inputs to model 1. Shear velocity assumed to be 500 m/sec. (*upper*) Resulting spectral ratios, normalized to surface motion obtained if no topography were present, at sites 1 and 10.

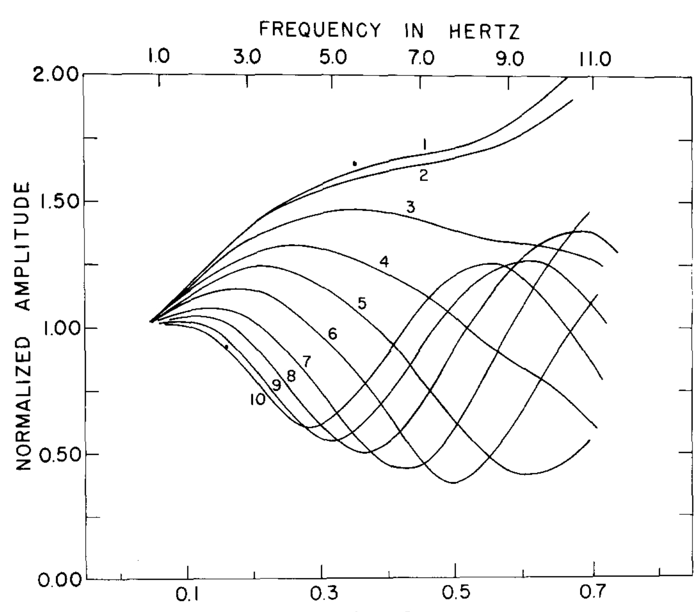

FIG. 5. Spectral ratio results for model 1 at various localities along the surface. Upper frequency scales represent assumed shear velocity of 500 m/sec.

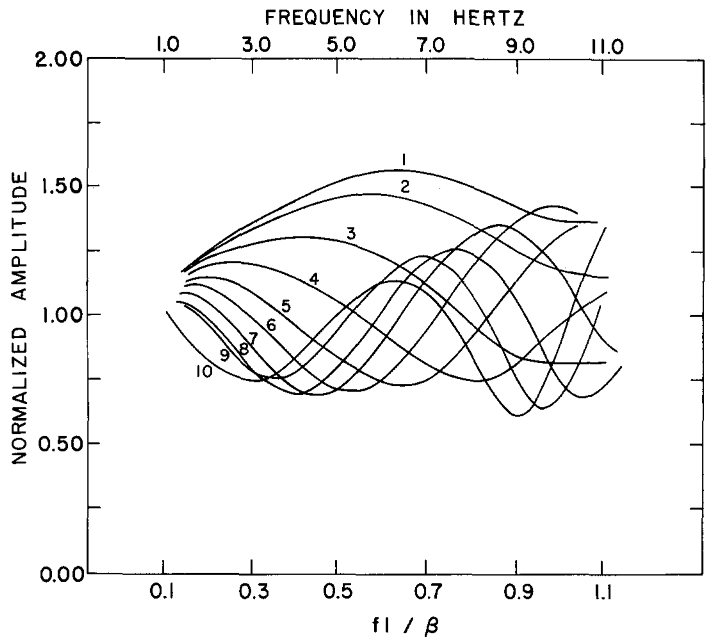

FIG. 6. Spectral ratio results for model 2.

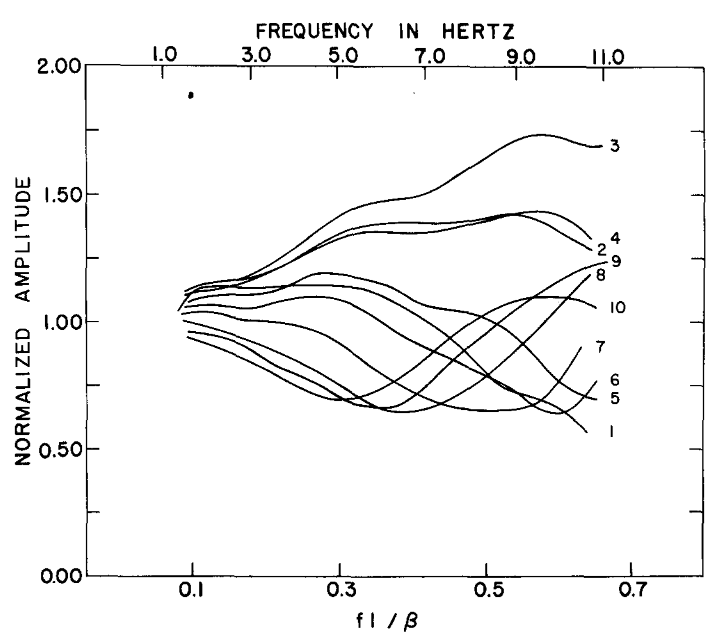

FIG. 7. Spectral ratio results for model 3.

In the computations, a velocity of 500 m/sec was assumed, but the results are unchanged if distances, velocities, and frequencies are scaled appropriately. This is emphasized in the figures by using nondimensional frequency for the bottom abscissas. The nondimensional frequency is given by $f\,l/\beta$, where f is frequency, β is shear velocity, and l is a characteristic scale length of the structure (here defined as the longest horizontal distance between the base and the top of the structure). These coordinates are also equal to the inverse of wavelength divided by the scale length. By using nondimensional frequency, a comparison of models 1 and 2 shows directly the effect of different height-to-width ratios on the spectral results. The shapes of the curves at equivalent sites are about the same, but the over-all variation from unity decreases as the slope decreases.

The results from all three models are quite similar. Each shows an amplification at the crest of the ridge and an oscillating amplification to deamplification pattern on the ridge flanks. Furthermore, the results tend to unity as the frequency decreases. Although in these models an amplification was always observed at the ridge crest, the complicated pattern on the ridge flanks cautions against making general conclusions about amplifications at ridge crests. In other models, and at angles of incidence other than vertical, deamplification could occur (Aki, 1971, oral communication). The important result here is that topography can cause significant amplifications (approaching 100 per cent) and can influence motion of surprisingly long wavelength (25 per cent amplification when $\lambda/l = 6$, where λ = wavelength and l = scale length, chosen as the *half-width* of the mountain in this case).

Discussion

Observational evidence of topographic effects. The calculations presented above indicate that topography-dependent amplifications and deamplifications can occur for *SH* waves. There is also field evidence for such effects on vertical accelerations. Probably the most reliable evidence occurs when the accelerations locally exceed that of gravity, for then the effects are of a more lasting nature than when the accelerations are lower. A few possible effects are regions of churned and overturned ground and boulders on horizontal ground thrown out of their sockets. Such observations have been made following the Indian earthquake of 1897 (Richter, 1958, p. 50–51), the Cedar Mountain earthquake of 1932 (Gianella and Callaghan, 1934, p. 2), the Hebgen Lake earthquake of 1959 (Hadley, 1964, p. 137), and the San Fernando earthquake of 1971 (Nason, 1971). The existence of accelerations exceeding 1 g does not necessarily imply an amplification of the waves, for there is no fundamental reason that the ambient acceleration cannot exceed the acceleration due to gravity. The reports of Hadley (1964) and Nason (1971), however, state a clear dependence of the amplification on topography. In particular, Hadley stated that all regions of churned earth ". . . . were found on ridge crests or other topographic eminences."

The calculations in this paper are based on *SH* motion and are, therefore, not directly applicable to the churned earth observations, which imply high vertical accelerations. One might think that the solution for vertically-incident *P* or *SV* would be similar to the *SH* solution, but reference to curves of *P-SV* scattering from a free surface (Ewing *et al.*, 1957) shows that for a *SV* wave at a 35° angle of incidence, the reflected *P* wave will be larger than the reflected *SV* wave. Thus, the *P-SV* interaction is quite strong, and the *SH* results may be very different from the *P-SV* results. It is instructive, however, to compare the *SH* results with conclusions drawn from field observations of topographic amplification of vertical accelerations. The calculations in this paper imply that the accelerations would have to be sufficient to produce accelerations of at least 0.5 g in the absence of topography in order to cause accelerations exceeding 1 g in regions of topographic relief. Furthermore, these accelerations would have to be of relatively high frequency (depending on the scale length of the topography and the velocity of the waves). Because high-frequency motion is subject to pronounced energy loss through scattering and material attenuation, favorable conditions would have to exist if the requirements above are to be met. In this regard, it is interesting to note Hadley's observation that "no areas underlain by thick soil, other surficial material, or even the weaker bedrock formations are known to have been churned . . ." This may imply that the conditions necessary for an amplification above 1 g were only just fulfilled. Nason's observations showed amplification on material which could not be described as extremely competent, but other observations in the region (e.g., Morrill, 1971) indicated that the basic accelerations could have been close to that of gravity and, thus, would require less favorable circumstances than in the Hebgen Lake earthquake for their amplification to values greater than 1 g.

Pacoima record—qualitative remarks: The Pacoima Dam strong-motion record, showing several high-frequency (approximately 10 Hz) peaks over 1 g, was recorded on an accelerograph located on a steep ridge in a region of pronounced relief (Trifunac and Hudson, 1971). The motion probably represents a complex mixture of contributions from different sectors of the rupture surface, which passed beneath the site and was such that a fairly large part of it was almost equidistant from the recording site. These body-wave motions cannot be simply described by a single, plane-wave incident from below. Furthermore, Trifunac (1971) has recently emphasized that surface waves may contribute significantly to the recorded ground acceleration. Since, however, the faulting had a strike-slip component, at least part of the horizontal motion recorded in the near-field should contain energy carried by *SH* waves (considering the approximate E-W strike of the fault plane, most of the *SH* energy should be on the S74 W component) and, thus, the calculations in this paper should be applicable to some of the recorded energy. Even if all of the energy resulted from vertically-incident, plane *SH* waves, it would be difficult to model the complex topography and, thus, predict quantitatively its effect on the motion. Notwithstanding these limitations, however, some general comments concerning the possible importance of topography are probably justified. The site is situated on top of a bench in the ridge, with a steep cliff face below it. Assuming a shear-wave velocity of 2 km/sec, the highest frequency components in the recorded acceleration, near 10 Hz, would have wavelengths of 200 m. This is several times larger than scale lengths associated with the local protuberance on which the accelerometer sits. From the calculations in this paper, it would seem that no unusual effects should arise from this local topography. On the other hand, the ridge on which the bench is located extends upward almost 500 m vertically in about 800 m, for an average slope of 30°. The scale lengths of this topography are on the order of the wavelengths of the predominant energy in the record (at ≈ 2 Hz), and, thus, we can say from the calculations in this paper that the over-all topography should produce a significant effect on the recorded energy. Whether the effect, at a given frequency, would be to amplify or deamplify the waves is impossible to say. We can estimate, however, that the difference between the recorded accelerations and those which would have been recorded if the topographic effects had not been present is uncertain to within at least 25 per cent at any period. The fact, however, that most of the spectral ratio curves in Figures 5, 6, and 7 for sites along the ridge flanks show an oscillating pattern of amplification and deamplification as a function of frequency could mean that the effect of topography on the over-all time series may not be as large as would be expected from the maximum deviation predicted for single frequencies.

CONCLUSIONS

The calculations in this paper and observations after several earthquakes of topographically-correlated amplifications of seismic energy show that the effect of topography can be quite important in determining the spatial distribution of accelerations. The calculations were for *SH* motion, which is an important component for earthquake engineering. The results lead to the observation that, although layering effects of low-velocity sediments are usually responsible for large amplifications, structures located on bedrock outcrops are not necessarily immune to high accelerations.

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APPENDIX

As noted before, the free surface was modeled by flat portions and 45° steps. In this way, every gridpoint was either on the free surface or an integral number of grid spacings removed. Thus, approximations to the derivatives could be made without using asymmetrical differences. As explained in previous papers, the free surface can be taken into account by using the boundary condition $\partial u/\partial n = 0$, where $\partial/\partial n$ indicates a normal derivative, to assign displacement values to a row of "fictitious" gridpoints immediately above the actual free surface. Knowing these values, the usual difference approximation to the equation of motion can be used up to, and including, grid points on the free surface to determine new values of displacements everywhere but along the fictitious row of points. The boundary condition is used to assign these values, and the whole process is then recycled. With reference to Figure 8, the fictitious point 1 was simply assigned the current value of the gridpoint beneath it indicated by the *dashed line*. This mirroring then produced an approximate zero normal derivative at the free surface. The same approach can be used when the surface is not horizontal. Points 2, 3, and 4 show the three types of gridpoint-boundary configurations possible with the type of surface discretization used here. In the calculations in this paper, these points were

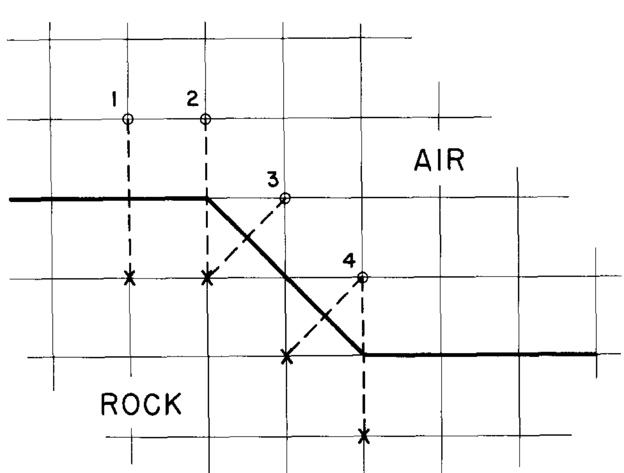

FIG. 8. Approximation of the continuum near a free surface. See text.

assigned values according to this scheme: point 2 was treated just as was point 1, point 3 was given the displacement value at the gridpoint connected to it by the *dashed line*, and point 4 was assigned $2/(2+\sqrt{2})$ times the value at the diagonal gridpoint plus $\sqrt{2}/(2+\sqrt{2})$ times the value of the gridpoint directly beneath and connected to it by the *dashed line*. The actual surface configuration implied by this scheme is probably a somewhat smoothed version of the discretized boundary. Furthermore, as discussed earlier, if the wavelength is long compared to the gridspacing, the jaggedness of the boundary does not affect the wave.

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