November 22, 2003, file: `c:\respspct\sd_for_directivity_pulses.tex`

SD for Directivity Pulses

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This is a brief note checking my assertion to Paul Spudich that the response of very long-period oscillators to displacement pulses such as occur for simple models of directivity in the forward and back azimuths do not approach one another, unlike the Fourier spectra.

To check this, I revised my program `smc_make.for` to create a smc file for a given number of cycles of a sinusoid, starting at a specified fraction of the sinusoid period. It turns out that a single cycle of a cosine in acceleration (given by running `smc_make` with one cycle and a start of 0.25 of the period) corresponds to a simple pulse in displacement (a raised cosine). Here are the equations:

$$a(t) = A \cos \frac{2\pi(t - T_1)}{T} \tag{1}$$

$$v(t) = A \frac{T}{2\pi} [\sin \frac{2\pi(t - T_1)}{T}] \tag{2}$$

$$d(t) = A (\frac{T}{2\pi})^2 [1 - \cos \frac{2\pi(t - T_1)}{T}] \tag{3}$$

where $T = T_2 - T_1$ and $T_1 \leq t \leq T_2$. From the last equation,

$$D = 2A(\frac{T}{2\pi})^2. \tag{4}$$

In addition, the area under the displacement is

$$\int_0^T d(t)dt = A(\frac{T}{2\pi})^2 T. \tag{5}$$

From this, the condition that the area of two displacement pulses be the same (as for simple directivity pusles) places the following contraints on the acceleration amplitude and duration:

$$A_2/A_1 = (\frac{T_1}{T_2})^3, \tag{6}$$

and in this case, the ratio of displacements becomes:

$$D_2/D_1 = \frac{T_1}{T_2}. \tag{7}$$

I contructed two acceleration time series, one with $T = 1$, $A = 1000$, and the other with $T = 10$ and $A = 1$. I used `smc2vd`, `smc2rs`, and `smc2fs` to compute velocity and displacement time series, displacement response spectra (for 5% damping), and Fourier spectra. The results are shown in the following graphs. The displacement pulses show be zero at the end of the acceleration pulse, but the $T = 1$ sec pulse shows a finite offset. This is probably due to roundoff in the double integration, and does not affect the results in any important way.

As expected, the relative displacement response spectra are constant for long oscillator periods, with levels equal to the peak displacements. The Fourier spectra, on the other had, approach one another at low frequencies, as they should.

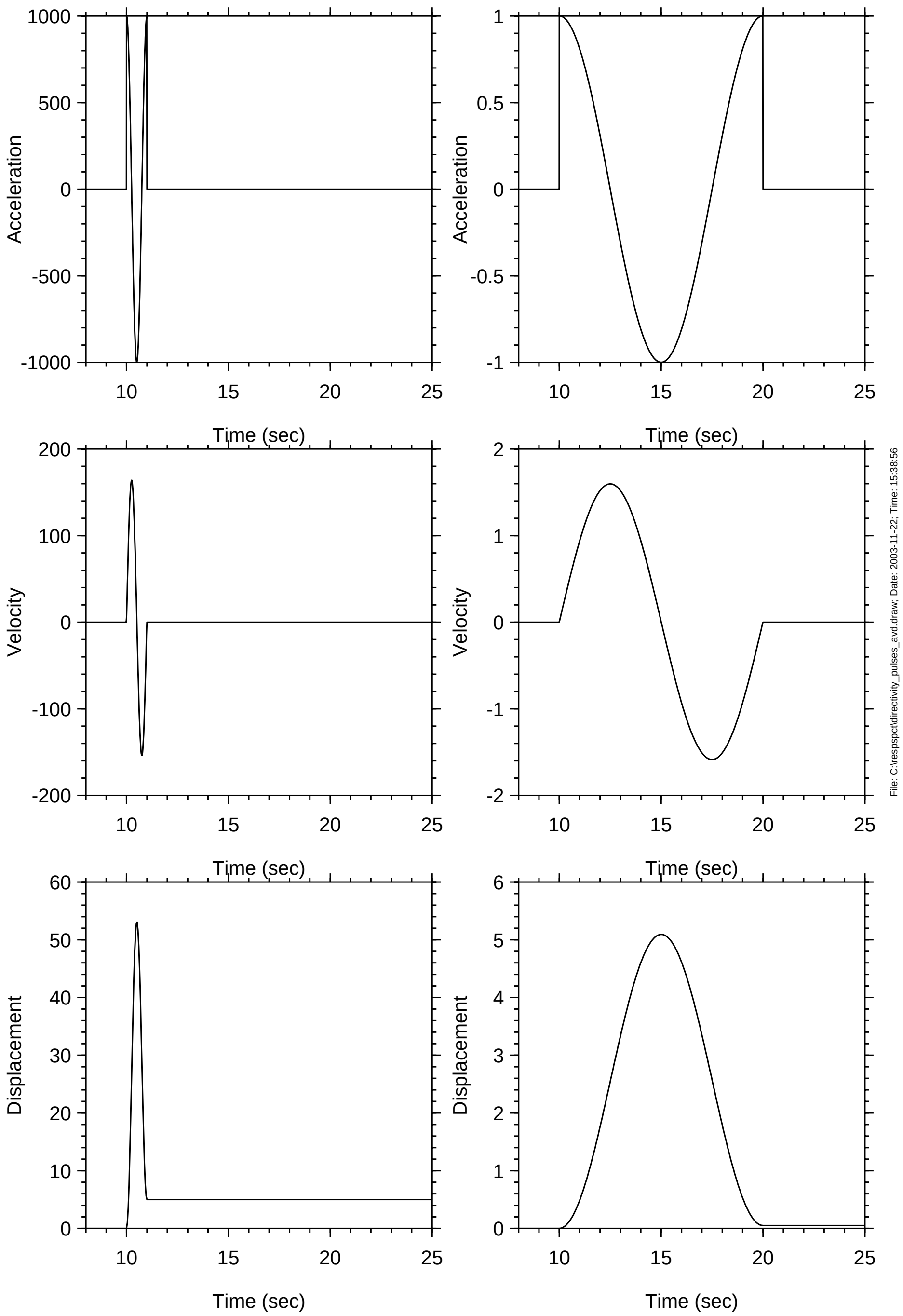

Figure 1. Acceleration, velocity, and displacements.

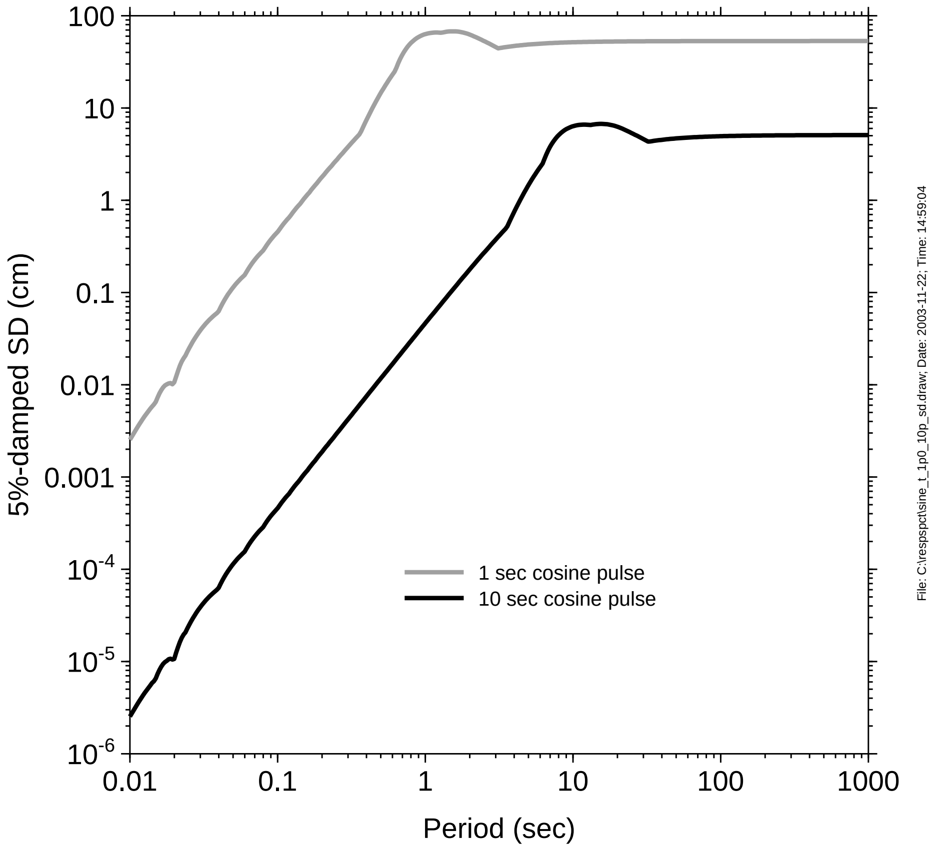

Figure 2. Response spectra for the two acceleration pulses shown in the previous figure.

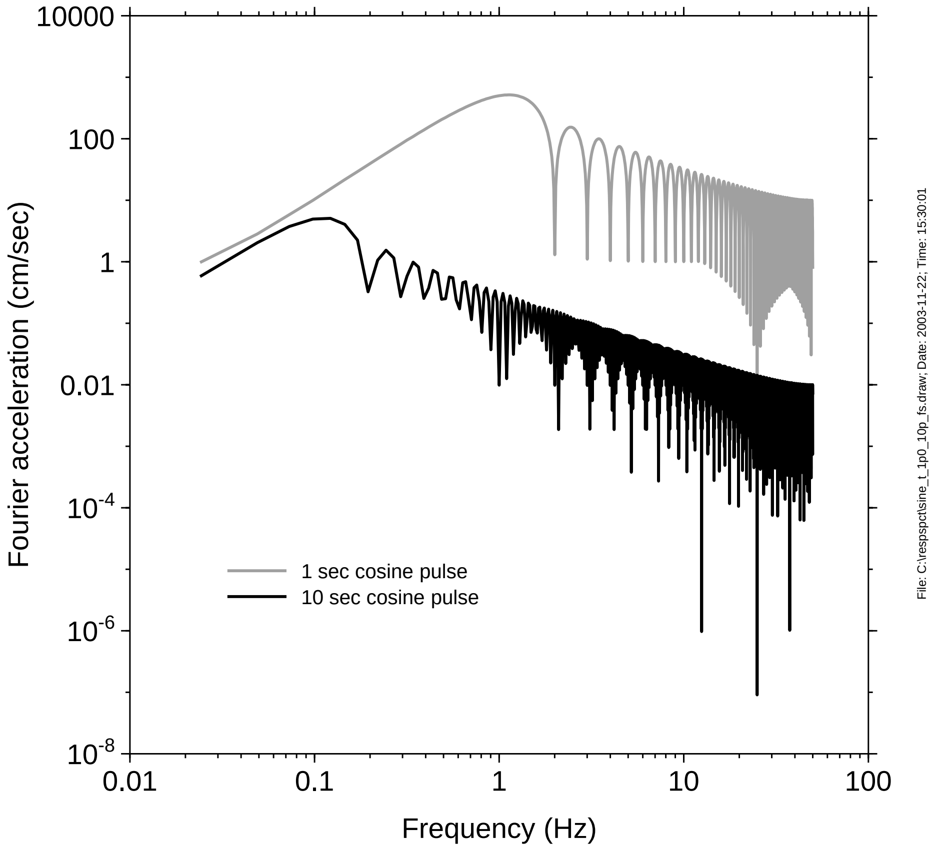

Figure 3. Fourier spectra for the two acceleration pulses shown in the previous figure.