A single function that produces a smooth transition between two $(R/R_{REF})^{\gamma}$ geometrical spreading functions

David M. Boore

In Stafford et al. (202x), the following single equation is used to represent a geometrical function that approaches $\ln g(R) = \gamma_1 \ln R$ for small R and $\ln g(R) = \gamma_f \ln R$ for large R, with a smooth transition at $R = r_t$ between these limits.

$$\ln g(R) = \gamma_1 \ln(R\,) - \frac{(\gamma_1 - \gamma_f)}{2} \ln\left(\frac{R^2 + r_t^2}{r_0^2 + r_t^2}\right) \tag{1}$$

(I have changed the signs of γ_1 and γ_f from that used in the equation in Stafford et al. so as to be consistent with the specification of the geometrical spreading in my SMSIM programs.)

To generalize:

$$\ln g(R) = \gamma_1 \ln(R/R_{REF}) - (\gamma_1 - \gamma_2)\frac{1}{\xi} \ln\left(\frac{R^{\xi} + R_T^{\xi}}{R_{REF}^{\xi} + R_T^{\xi}}\right) \tag{2}$$

or better (easier to see asymptotic values)

$$\ln g(R) = \gamma_1 \ln(R/R_{REF}) - (\gamma_1 - \gamma_2)\frac{1}{\xi} \ln\left(\frac{(R/R_{REF})^{\xi} + (R_T/R_{REF})^{\xi}}{1 + (R_T/R_{REF})^{\xi}}\right) \tag{3}$$

where the rate of transition from γ_1 to γ_2 around the transition distance R_T is controlled by ξ. R_{REF} is the distance at which $g = 1$.

Larger ξ gives a sharper transition (e.g., $\xi = 10$ produces a spreading similar to a bilinear spreading). To make this more apparent, here is an equation for ξ in terms of the ratio $g_{RAT} = g(R_T)/g_1(R_T)$, where $g_1(R) = (R_{REF}/R)^{\gamma_1}$:

$$\xi = (\gamma_2 - \gamma_1) \ln \frac{2}{\ln g_{RAT}} \tag{4}$$

and

$$g_{RAT} = \exp\left(\frac{\gamma_2 - \gamma_1}{\xi} \ln 2\right) \tag{5}$$

Substituting equation (4) into equation (3) gives:

$$\ln g(R) = \gamma_1 \ln(R/R_{REF}) + \frac{\ln g_{RAT}}{\ln 2} \ln\left(\frac{(R/R_{REF})^{\xi} + (R_T/R_{REF})^{\xi}}{1 + (R_T/R_{REF})^{\xi}}\right) \tag{6}$$

The actual ratio $g(R_T)/g_1(R_T)$ resulting from ξ given by equation (4) asymptotically approaches g_{RAT} for $R_T/R_{REF} \gg 1$. But for practical purposes, when R_T is several tens of km and $R_{REF} = 1$, the actual ratio is very close to the asymptotic value. With $\gamma_1 = -1.0$, $\gamma_2 = -0.5$, and $\xi = 2$, equation (5) gives $g_{RAT} = 1.189$. That value is used in the graphs below.

Here is an example with $\gamma_1 = -1.0$, $\gamma_2 = -0.5$, $R_T = 40$ km, $R_{REF} = 1$ km, and two values of g_{RAT}: 1.189 and 1.035. The graph below plots *gsprd* from the SMSIM program *fmrsk_loop_fas_drvr*, along with a direct evaluation of the function for $g_{RAT} = 1.035$ (corresponding to $\xi = 10$). For comparison, the two-segment standard *gsprd* function is also shown. The $g_{RAT} = 1.035$ results are almost the same as the standard, two segment *gsprd* function. As a check of the program *fmrsk_loop_fas_drvr,* the spreading is shown for two values of oscillator period. As the figure shows, the spreading is independent of period (as it should be).

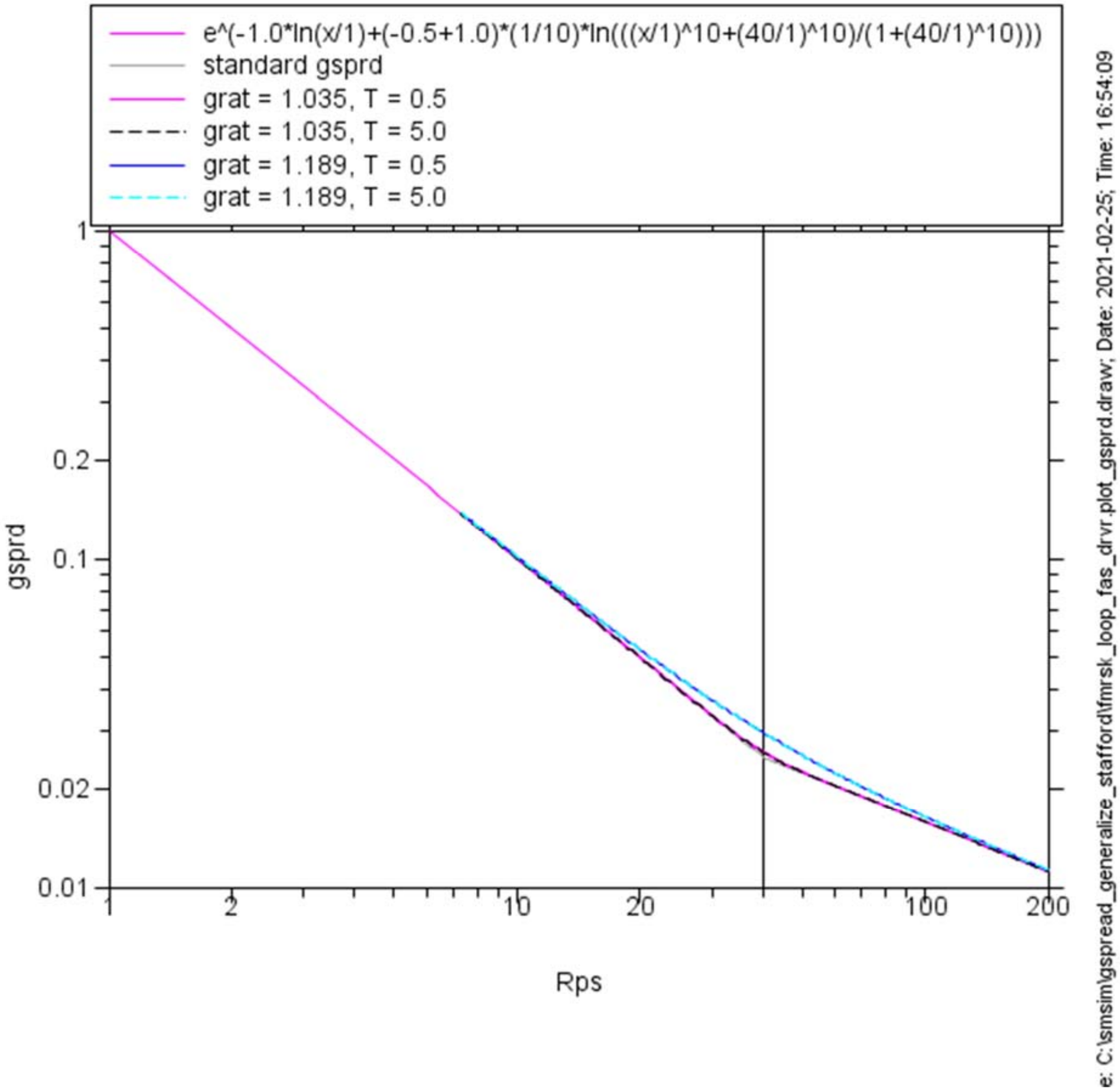

Figure 01.

An expanded version near the transition distance is shown in Figure 02.

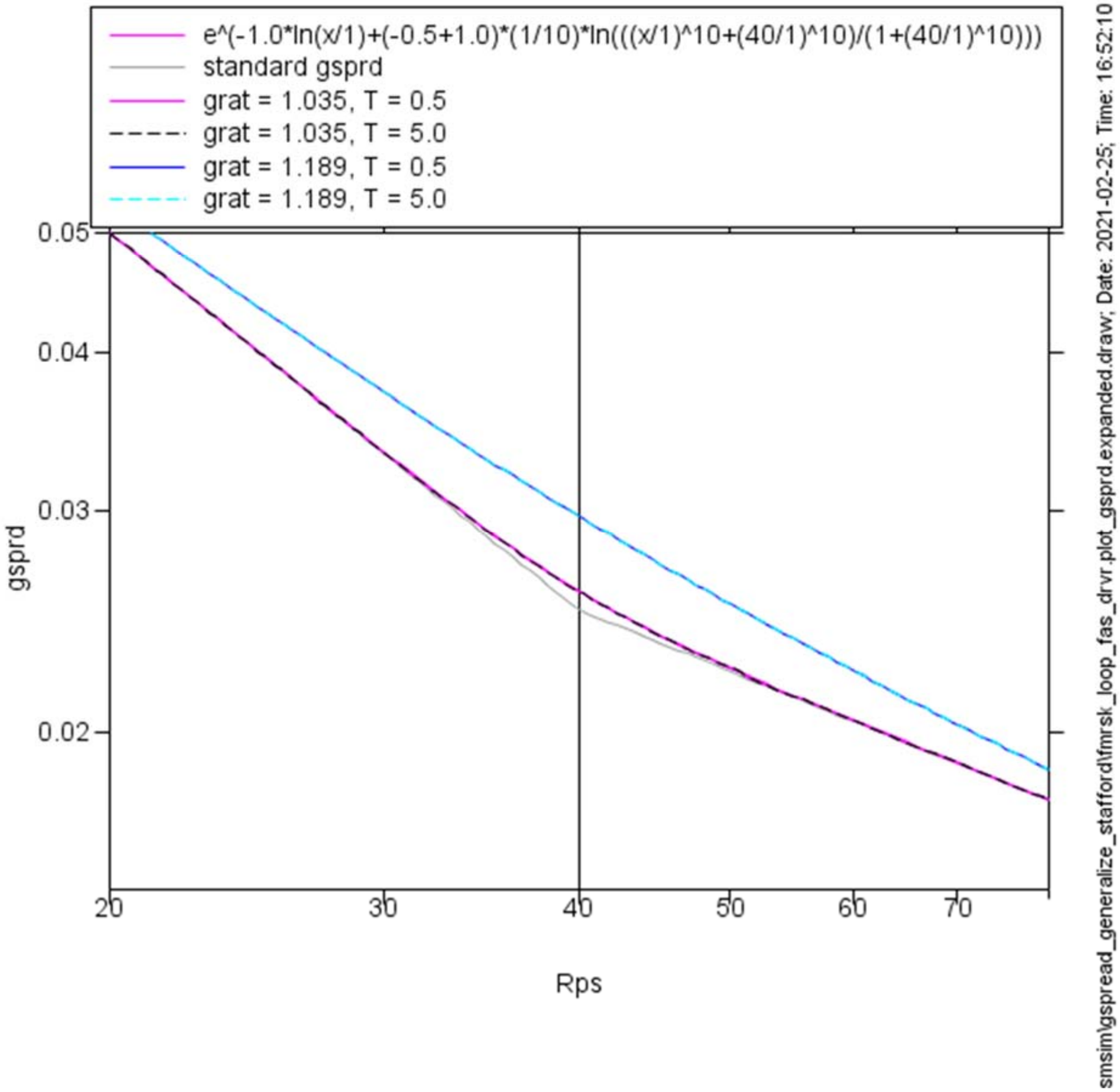

Figure 02. An expanded version of Figure 01, showing more detail near the transition.

The two graphs show that the function is working properly.

References

Stafford, P. J., D. M. Boore, R. R. Youngs, and J. J. Bommer (202x). Host-region parameters for an adjustable model for crustal earthquakes: facilitating the implementation of the backbone approach to building ground-motion logic trees in PSHA, *Earthquake Spectra*, submitted.