Notes on ratios of source spectra

Dave Boore

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These notes are concerned with the function describing the ratio of two Fourier spectra, as might be computed in empirical Green's function analyzes of a mainshock and one of its aftershocks, the goal being to determine the stress parameter of the mainshock (and secondarily, the aftershock) without making assumptions about geometrical spreading or site response.

Consider the ratio of two single-corner-frequency ω^{-2} source spectra $S_j = M_0 \Big/ \left[1 \big/ 1 + \left(f / f_{cj} \right)^2 \right]$, where j = 1, 2.

$$\frac{S_1}{S_2} = \left(\frac{M_{01}}{M_{02}} \right) \left(\frac{1 + \left(f / f_{c2} \right)^2}{1 + \left(f / f_{c1} \right)^2} \right) \quad (1)$$

Define the low-frequency intercept as *LF* and the high-frequency intercept as *HF*. Then

$$LF = \frac{M_{01}}{M_{02}} \quad (2)$$

and

$$HF = \left(\frac{M_{01}}{M_{02}} \right) \left(\frac{f_{c1}}{f_{c2}} \right)^2 . \quad (3)$$

As a side note, note that given *LF* and the equation defining the moment magnitude (for which $\log M_0 \sim 1.5\mathbf{M}$), the difference of moment magnitudes is given by

$$\mathrm{M}_1 - \mathrm{M}_2 = \frac{2}{3} \log \left(LF \right) \quad (4)$$

With the relations in equations (1), (2), and (3), the ratio of spectra for two events can be written in this form, which is convenient when fitting a curve manually to observed ratios, e.g., using a function in CoPlot:

$$\frac{S_1}{S_2} = LF\left(\frac{1+\left(\frac{HF}{LF}\right)\left(\frac{f}{f_{c1}}\right)^2}{1+\left(\frac{f}{f_{c1}}\right)^2}\right) \tag{5}$$

This is convenient because the three parameters, *LF*, *HF*, and f_{c1}, can be estimated visually from the observed ratios, whereas it is more difficult to estimate f_{c2}.

Given these three parameters, the second corner frequency is given by:

$$f_{c2} = f_{c1}\sqrt{\frac{LF}{HF}} \tag{6}$$

So far no assumption has been made about the stress parameters of the two events. Using the standard relation

$$M_0 f_c^3 \sim \Delta\sigma \tag{7}$$

Gives

$$\frac{\Delta\sigma_1}{\Delta\sigma_2} = HF\sqrt{\frac{HF}{LF}}\ . \tag{8}$$

Equality of the stress parameters requires

$$HF = LF^{1/3}. \tag{9}$$

or

$$LF = HF^3 \tag{10}$$

The stress parameter is given as a function of the seismic moment M_0, the source corner frequency f_c, and the shear-wave velocity in the vicinity of the source β_s by the relation

$$\Delta\sigma = 8.47 M_0 \left(\frac{f_c}{\beta_s} \right)^3 \tag{11}$$

where all quantities are in the same system of units; in terms of the usual mixed units (stress in bars, moment in dyne-cm, shear-wave velocity in km/s), this becomes

$$\Delta\sigma = 8.47 \times 10^{-21} M_0 \left(\frac{f_c}{\beta_s} \right)^3 . \tag{12}$$

(As another side note, brought up by Ralph Archuleta's comments during the 11-13 October 2011 PEER NGA-East workshop, the relation used by Allmann and Shearer (2009) is

$$\Delta\sigma = 13.5 M_0 \left(\frac{f_c}{\beta_s} \right)^3 . \tag{13}$$

This equation is based on relations by Eshelby and Madariaga. The stress parameters derived for the same moment and corner frequency will differ by a factor of 1.6.)

References

Allmann, B. P. and P. M. Shearer (2009). Global variations of stress drop for moderate to large earthquakes, *J. Geophys. Res.* **114**, B01310, doi:10.1029/2008JB005821, 22 pp.