

**Notes on program *wholespace_pointsource*.for for
computing the complete wave field in a whole space
for a point-source dislocation.**

By David M. Boore

Version notes: 08 July 2018 – renamed the program from “wholsp_ptsrc” to “wholespace_pointsource”, but the program and the contents of this document are unchanged.

The program evaluates the following integral (slightly modified from equation 4.30 in Aki and Richards, 2002):

$$\begin{aligned}
 u_n = & \left(\frac{30\gamma_n\gamma_p\gamma_q\nu_q - 6\nu_n\gamma_p - 6\delta_{np}\gamma_q\nu_q}{4\pi\rho r^4} \right) M_0 \int_{r/\alpha}^{r/\beta} \tau h_p(t - \tau) d\tau \\
 & + \left(\frac{12\gamma_n\gamma_p\gamma_q\nu_q - 2\nu_n\gamma_p - 2\delta_{np}\gamma_q\nu_q}{4\pi\rho\alpha^2 r^2} \right) M_0 h_p\left(t - \frac{r}{\alpha}\right) \\
 & - \left(\frac{12\gamma_n\gamma_p\gamma_q\nu_q - 3\nu_n\gamma_p - 3\delta_{np}\gamma_q\nu_q}{4\pi\rho\beta^2 r^2} \right) M_0 h_p\left(t - \frac{r}{\beta}\right) \\
 & + \frac{2\gamma_n\gamma_p\gamma_q\nu_q}{4\pi\rho\alpha^3 r} M_0 \dot{h}_p\left(t - \frac{r}{\alpha}\right) - \left(\frac{2\gamma_n\gamma_p\gamma_q\nu_q - \nu_n\gamma_p - \delta_{np}\gamma_q\nu_q}{4\pi\rho\beta^3 r} \right) M_0 \dot{h}_p\left(t - \frac{r}{\beta}\right)
 \end{aligned}$$

where u_n is the n th component of the displacement vector, and repeated subscripts imply summation over the three components of each vector. γ and ν are unit vectors in the direction of the ray leaving the source and the fault normal, respectively (see the figure of geometry given on a later page). \mathbf{h} is the dislocation slip vector, normalized to have unit maximum amplitude. M_0 is the seismic moment. Assuming that the time dependence is the same for each component, the normalized slip vector can be written as

$$\mathbf{h} = h(t)\mathbf{i}_h$$

Following Joyner and Spudich's (1994) notation, the vector displacement \mathbf{u} equation above can be written as

$$\mathbf{u} = \mathbf{u}^N + \mathbf{u}^{IP} + \mathbf{u}^{IS} + \mathbf{u}^{FP} + \mathbf{u}^{FS}$$

where the individual terms represent the near-field, intermediate-field *P*-wave, intermediate-field *S*-wave, far-field *P*-wave, and far-field *S*-wave terms, as indicated by the superscripts.

Using a Cartesian coordinate system with x_1 , x_2 , x_3 = north, east, and down, the unit vectors are given by the equations implied by these Fortran subroutines:

```
* -----  
  subroutine unit_vector_p(i_p, az, toa)  
* Dates: 04/20/04 - Written by D. Boore  
  
  real i_p(3)  
  
  pi = 4.0*atan(1.0)  
  dtor = pi/180.0  
  
  azr = az*dtor  
  toar = toa*dtor  
  
  cos_az = cos(azr)  
  sin_az = sin(azr)  
  
  cos_toa = cos(toar)  
  sin_toa = sin(toar)  
  
  i_p(1) = + sin_toa*cos_az  
  i_p(2) = + sin_toa*sin_az  
  i_p(3) = + cos_toa  
  
  return  
  end  
* -----
```

The subroutine above gives the direction unit vector of the ray as it leaves the source with an azimuth of *az* clockwise from north and a take-off angle *toa*, measured from the downward vertical direction.

```
* -----  
  subroutine unit_vector_slip(i_slip, sa, da, ra)  
* Dates: 04/20/04 - Written by D. Boore  
  
  real i_slip(3)  
  
  pi = 4.0*atan(1.0)  
  dtor = pi/180.0  
  
  sar = sa*dtor  
  dar = da*dtor  
  rar = ra*dtor
```

```

cos_sa = cos(sar)
sin_sa = sin(sar)

cos_da = cos(dar)
sin_da = sin(dar)

cos_ra = cos(rar)
sin_ra = sin(rar)

i_slip(1) = cos_ra*cos_sa + cos_da*sin_ra*sin_sa
i_slip(2) = cos_ra*sin_sa - cos_da*sin_ra*cos_sa
i_slip(3) =                - sin_da*sin_ra

return
end
* -----
* -----
subroutine unit_vector_fn(i_fn, sa, da, ra)
* Dates: 04/20/04 - Written by D. Boore

real i_fn(3)

pi = 4.0*atan(1.0)
dtor = pi/180.0

sar = sa*dtor
dar = da*dtor
rar = ra*dtor

cos_sa = cos(sar)
sin_sa = sin(sar)

cos_da = cos(dar)
sin_da = sin(dar)

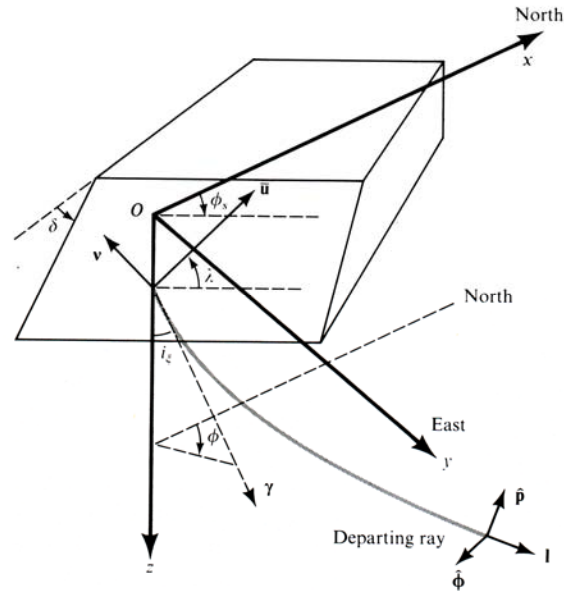
cos_ra = cos(rar)
sin_ra = sin(rar)

i_fn(1) = - sin_da*sin_sa
i_fn(2) = + sin_da*cos_sa
i_fn(3) = - cos_da

return
end
* -----

```

In these subroutines, the strike azimuth (*sa*), dip angle (*da*), and rake angle (*ra*) are as defined in Figure 4.20 of Aki and Richards (2002), shown below:



I use Joyner and Spudich's trick for evaluating the near-field term. Representing the integral in the near-field terms as

$$I = \int_{r/\alpha}^{r/\beta} \tau h(t - \tau) d\tau$$

the acceleration requires \ddot{i} . This is

$$\ddot{i} = h(t - t_\alpha) - h(t - t_\beta) + t_\alpha \dot{h}(t - t_\alpha) - t_\beta \dot{h}(t - t_\beta),$$

where t_α and t_β are travel times for P waves and S waves.

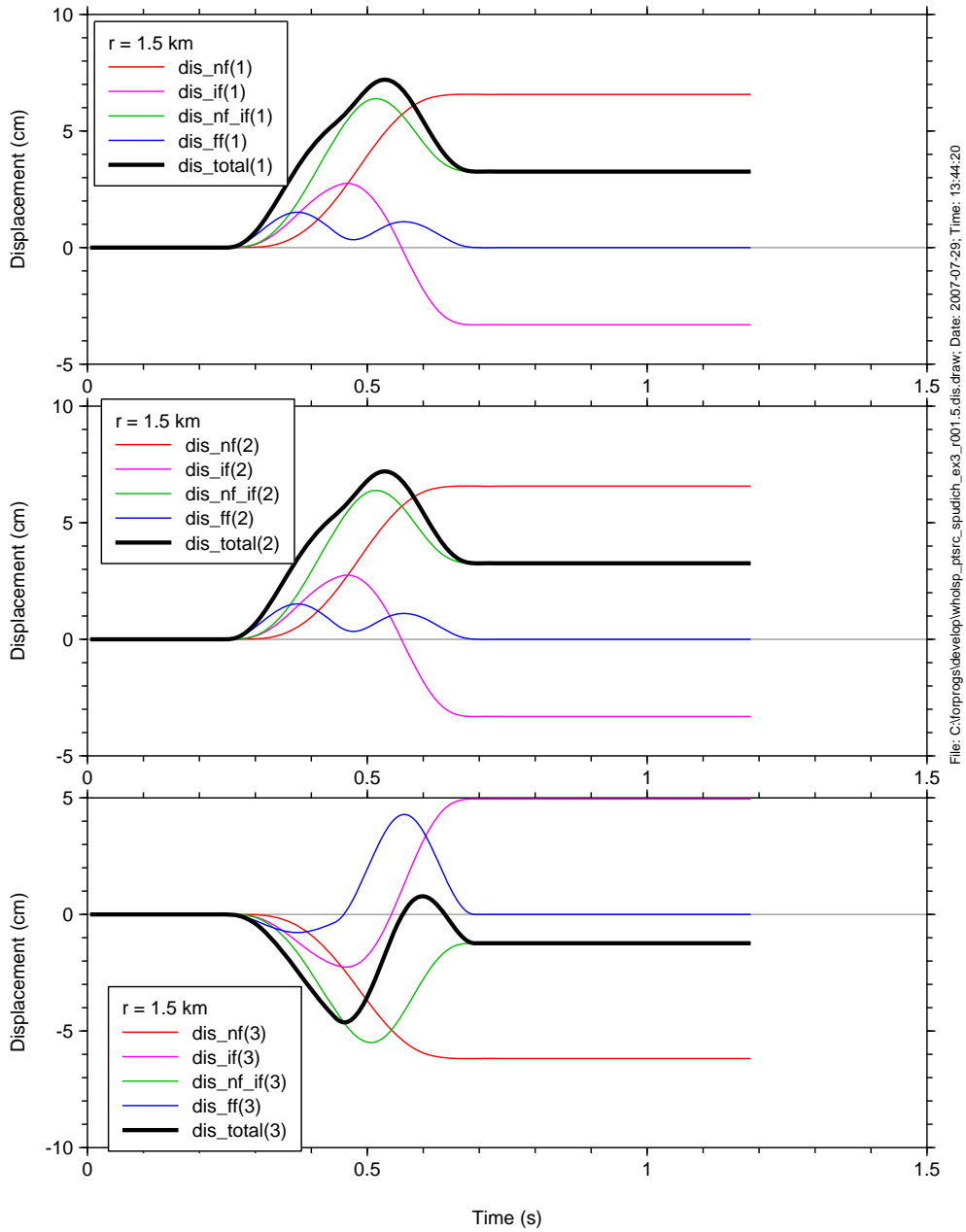
Thus, ground acceleration requires $h(t)$, $\dot{h}(t)$, $\ddot{h}(t)$, and $\ddot{\ddot{h}}(t)$. In addition, ground velocity and ground displacement require the first and second integrals of $h(t)$. For convenience in deriving analytical expressions for the derivatives and integrals of $h(t)$, I use the following equation for $h(t)$:

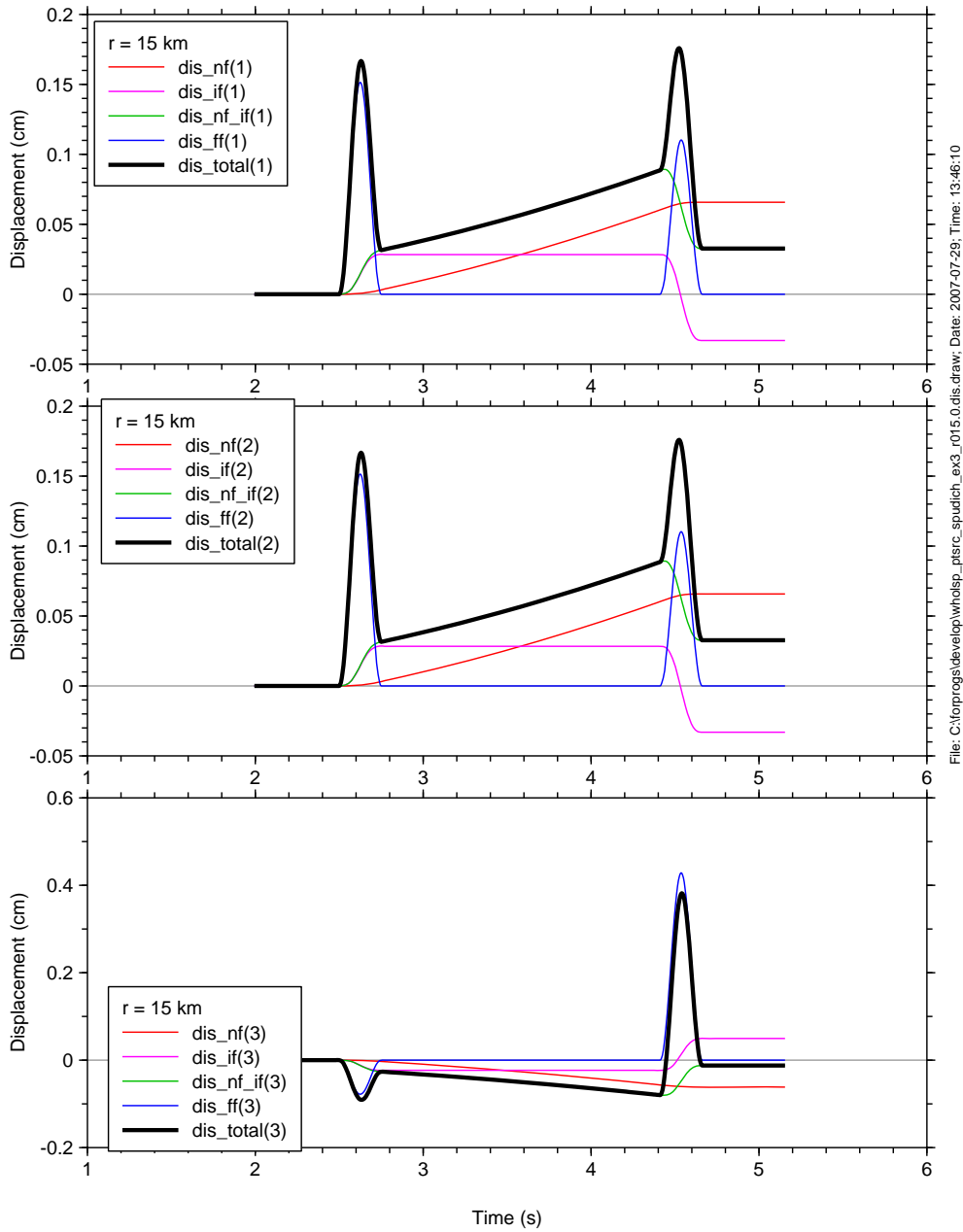
$$h(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{T} \left[t - \frac{T}{2\pi} \sin(2\pi t / T) \right], & 0 \leq t < T \\ 1, & T \leq t \end{cases}$$

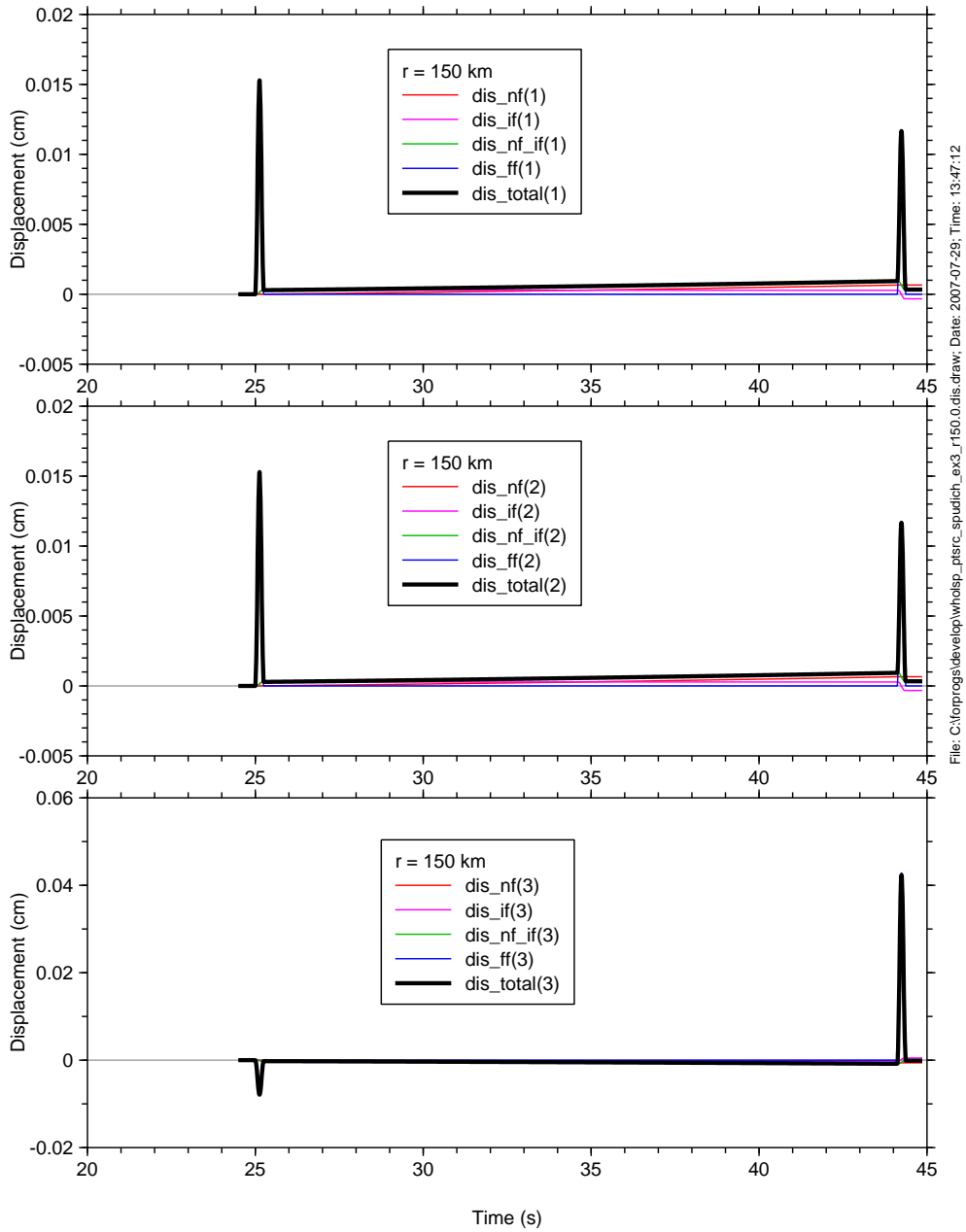
This form of the displacement was used by Makris and Chang (2000). For acceleration it corresponds to a single cycle of a sine wave. It is

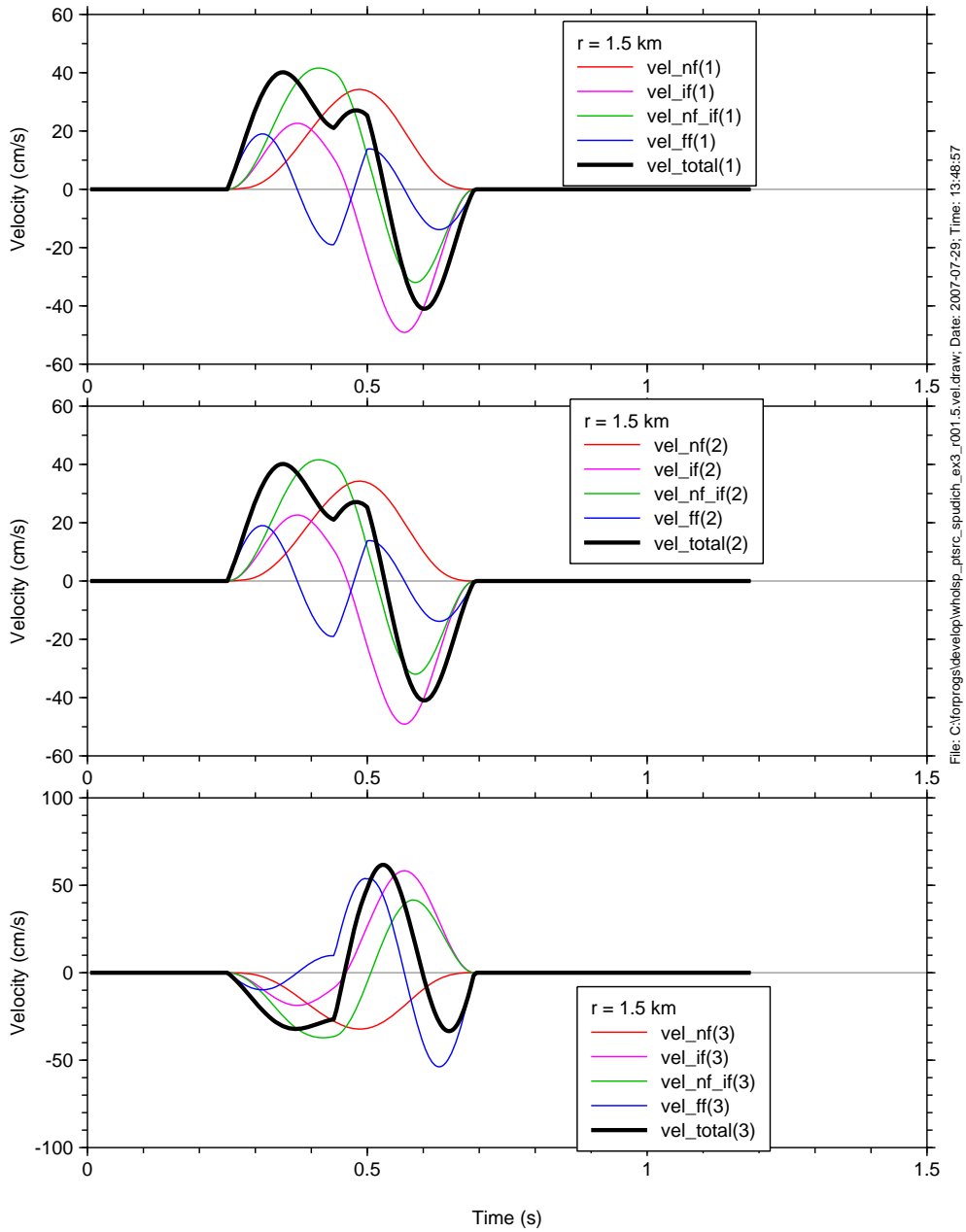
easy to differentiate and integrate this function analytically, and thus it is easy to write a program to evaluate the complete wavefield for a point source dislocation in a whole space. My program for doing this is *wholsp_ptsrc*; a listing of the program follows the references. Here are examples of the waveform for a particular case, for three components and displacements and velocities at three distances. The individual contributions of the near-, intermediate, and far-field terms are shown (as well as the sum of the near- and intermediate field terms). The control file used for these calculations is given here (for a distance of 1.5 km):

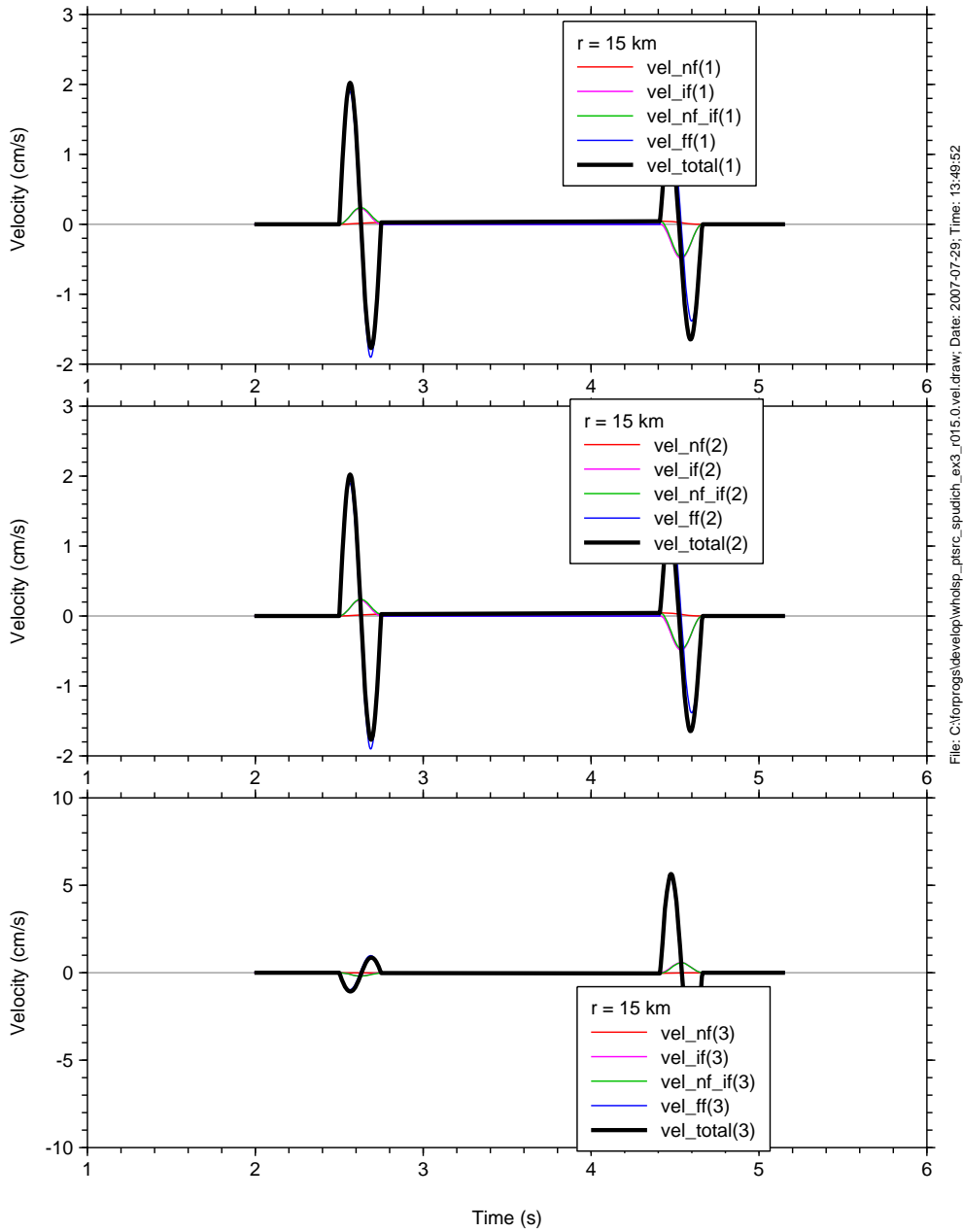
```
! Control file for program WholSp_PtSrc
!  
!Enter Vs, Vp, dens:  
    3.4 6.0 2.7  
!Enter sa,da,ra(degrees):  
    0 90 0  
!Enter az2sta, r2sta, toa, tp_minus_tstart,  
tend_minus_ts_plus_pw, dt:  
    45.0 1.5 110.0 0.5 0.5 0.005  
!Enter M, pulse width  
    5.0 0.25  
!Enter stem name of output file:  
    wholsp_ptsrc_spudich_ex3
```

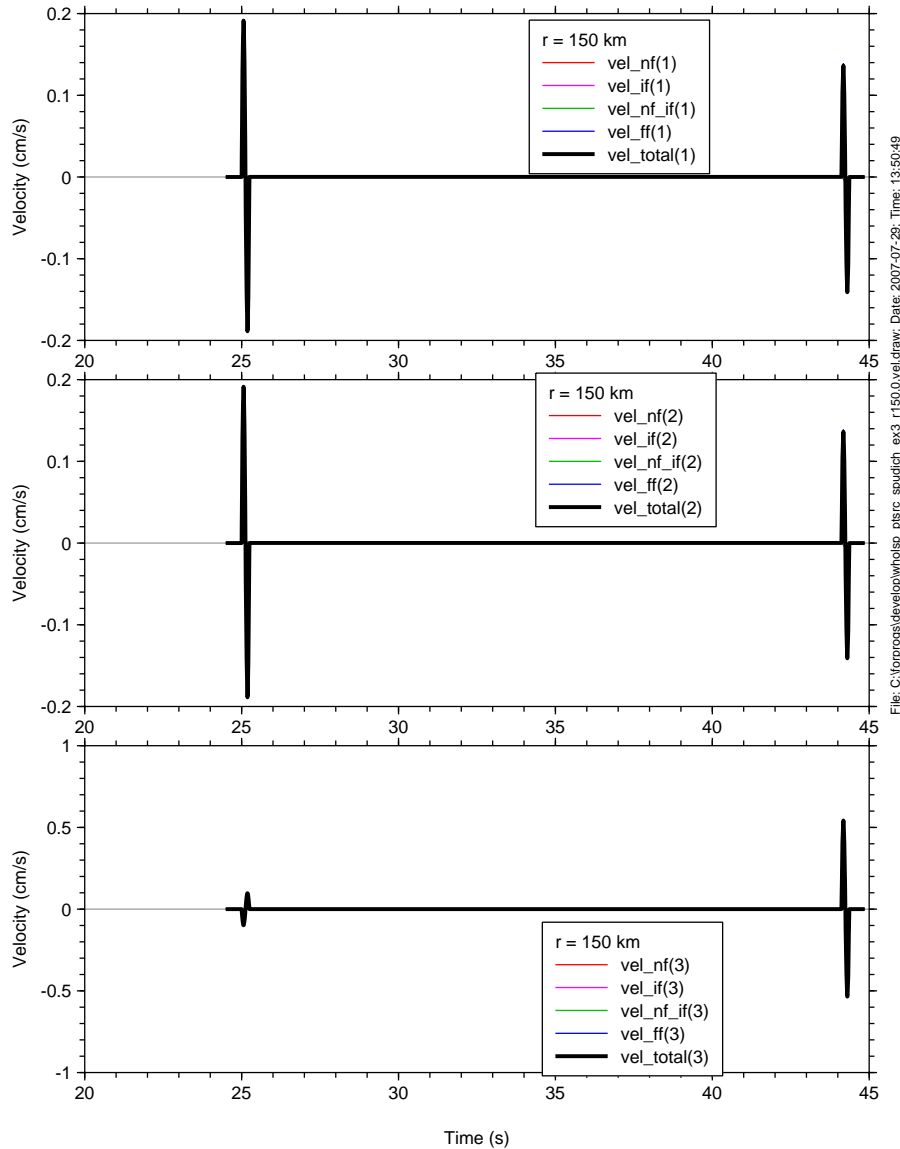












References

Aki, K. and P. G. Richards (2002). *Quantitative Seismology: Second Edition*, 700 p., University Science Books, Sausalito, California.

Joyner, W. B., and P. Spudich (1994). Including near-field terms in the isochrone integration method for application to finite-fault or Kirchoff boundary integral problems, *Bull. Seism. Soc. Am.* **84**, 1260—1265.

Makris, N., and S.-P. Chang (2000). Effect of viscous, viscoplastic, and friction damping on the response of seismic isolated structures, *Earthquake Engineering and Structural Dynamics* **29**, 85--107.


```

* -----
  Program Wholespace_Pointsource

* Compute seismograms for a point source in a whole space, including
* near-, intermediate-, and far-field terms. The assumed slip is
* equivalent to a single cycle of a sine wave.

*! Control file for program Wholespace_Pointsource
*!
*!Enter Vs, Vp, dens:
*   3.5 6.1 2.7
*!Enter sa,da,ra(degrees):
*   0.0 90.0 0.0
*!Enter az2sta, r2sta, toa, tp_minus_tstart, tend_minus_ts_plus_pw, dt:
*   20.0 10.0 90.0 5.0 5.0 0.05
*!Enter M, pulse width
*   7.0 10.0
*!Enter stem name of output file:
*   test_Wholespace_Pointsource

* Dates: 07/20/07 - Written by D. Boore, using equations in
*             Aki and Richards (text) and Joyner and Spudich
(BSSA)
!   07/08/18 - Changed name to Wholespace_Pointsource, bring in
util subroutines
!             in a single file made during compile time by
calling
!             make_wholespace_pointsource_util_subs_file.bat

!   implicit real*8 (a - h, o - z)

!   real*4 acc(*), omega, damp, dt, rd, rv,aa

character ctl_cmmnts(100)*79
character f_ctl*100, f_stem*100,
:         f_acc*100, f_vel*100, f_dis*100

logical f_ctl_exist
real i_slip(3), i_fn(3), i_p(3)
real radpat_nf(3), radpat_if_p(3), radpat_if_s(3),
:         radpat_ff_p(3), radpat_ff_s(3)
real acc_nf(3), acc_if(3), acc_nf_if(3), acc_ff(3), acc_total(3)
real vel_nf(3), vel_if(3), vel_nf_if(3), vel_ff(3), vel_total(3)
real dis_nf(3), dis_if(3), dis_nf_if(3), dis_ff(3), dis_total(3)

pi = 4.0*atan(1.)

f_ctl_exist = .false.
do while (.not. f_ctl_exist)
  f_ctl = ' '
  write(*, '(a\)\')

```

```

:      ' Enter name of control file '//
:      '(cr = wholespace_pointsource.ctl): '
      read(*, '(a)') f_ctl
      if (f_ctl .eq. ' ') f_ctl = 'wholespace_pointsource.ctl'
      call trim_c(f_ctl,nc_f_ctl)
      inquire(file=f_ctl(1:nc_f_ctl), exist=f_ctl_exist)
      if (.not. f_ctl_exist) then
        write(*,'(a)') ' ***** FILE DOES NOT EXIST ***** '
      end if
end do

call get_lun(nu_ctl)
open(unit=nu_ctl,file=f_ctl(1:nc_f_ctl),status='unknown')

call skipcmnt(nu_ctl,ctl_cmnts, nc_ctl_cmnts)
read(nu_ctl,*) vs, vp, dens

call skipcmnt(nu_ctl,ctl_cmnts, nc_ctl_cmnts)
read(nu_ctl,*) sa, da, ra

dd = sa + 90.0

call skipcmnt(nu_ctl,ctl_cmnts, nc_ctl_cmnts)
read(nu_ctl,*) az2sta, r2sta, toa,
:      tp_minus_tstart, tend_minus_ts_plus_pw, dt

call skipcmnt(nu_ctl,ctl_cmnts, nc_ctl_cmnts)
read(nu_ctl,*) amag, pw

amom = 10.0**(1.5*amag+16.05)

call skipcmnt(nu_ctl,ctl_cmnts, nc_ctl_cmnts)
f_stem = ' '
read(nu_ctl,'(a)') f_stem

r2stacm = r2sta * 1.0e+05
vpcgs = vp * 1.0e+05
vscgs = vs * 1.0e+05

tp = r2sta/vp
ts = r2sta/vs

afctr_nf = amom/(4.0 * pi * dens * r2stacm**4)
afctr_if_p = amom/(4.0 * pi * dens * vpcgs**2 * r2stacm**2)
afctr_if_s = amom/(4.0 * pi * dens * vscgs**2 * r2stacm**2)
afctr_ff_p = amom/(4.0 * pi * dens * vpcgs**3 * r2stacm)
afctr_ff_s = amom/(4.0 * pi * dens * vscgs**3 * r2stacm)

call unit_vector_slip(i_slip, sa, da, ra)
call unit_vector_fn(i_fn, sa, da, ra)
call unit_vector_p(i_p, az2sta, toa)

do i = 1, 3

  radpat_nf(i) =
:      30 *      i_p(i) * dot(i_p,i_fn,3) * dot(i_p,i_slip,3)

```

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:           - 6 *   i_fn(i) * dot(i_p,i_slip,3)
:           - 6 * i_slip(i) * dot(i_p,i_fn,3)

  radpat_if_p(i) =
:           12 *   i_p(i) * dot(i_p,i_fn,3) * dot(i_p,i_slip,3)
:           - 2 *   i_fn(i) * dot(i_p,i_slip,3)
:           - 2 * i_slip(i) * dot(i_p,i_fn,3)

  radpat_if_s(i) =
:           - 12 *   i_p(i) * dot(i_p,i_fn,3) * dot(i_p,i_slip,3)
:           + 3 *   i_fn(i) * dot(i_p,i_slip,3)
:           + 3 * i_slip(i) * dot(i_p,i_fn,3)

  radpat_ff_p(i) =
:           2 *   i_p(i) * dot(i_p,i_fn,3) * dot(i_p,i_slip,3)

  radpat_ff_s(i) =
:           - 2 *   i_p(i) * dot(i_p,i_fn,3) * dot(i_p,i_slip,3)
:           +   i_fn(i) * dot(i_p,i_slip,3)
:           +   i_slip(i) * dot(i_p,i_fn,3)

end do

call trim_c(f_stem,nc_f_stem)

f_acc = ' '
f_acc = f_stem(1:nc_f_stem)//'.acc.out'
call trim_c(f_acc, nc_f_acc)

call get_lun(nu_acc)
open(unit=nu_acc,file=f_acc(1:nc_f_acc),status='unknown')

write(nu_acc,'(5x,a, 3(4x,a, 4x,a, 1x,a, 4x,a, 1x,a))')
:           'time',
: 'acc_nf(1)', 'acc_if(1)', 'acc_nf_if(1)',
: 'acc_ff(1)', 'acc_total(1)',
: 'acc_nf(2)', 'acc_if(2)', 'acc_nf_if(2)',
: 'acc_ff(2)', 'acc_total(2)',
: 'acc_nf(3)', 'acc_if(3)', 'acc_nf_if(3)',
: 'acc_ff(3)', 'acc_total(3)'

f_vel = ' '
f_vel = f_stem(1:nc_f_stem)//'.vel.out'
call trim_c(f_vel, nc_f_vel)

call get_lun(nu_vel)
open(unit=nu_vel,file=f_vel(1:nc_f_vel),status='unknown')

write(nu_vel,'(5x,a, 3(4x,a, 4x,a, 1x,a, 4x,a, 1x,a))')
:           'time',
: 'vel_nf(1)', 'vel_if(1)', 'vel_nf_if(1)',
: 'vel_ff(1)', 'vel_total(1)',
: 'vel_nf(2)', 'vel_if(2)', 'vel_nf_if(2)',
: 'vel_ff(2)', 'vel_total(2)',

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: 'vel_nf(3)', 'vel_if(3)', 'vel_nf_if(3)',
: 'vel_ff(3)', 'vel_total(3)'

f_dis = ' '
f_dis = f_stem(1:nc_f_stem)//'.dis.out'
call trim_c(f_dis, nc_f_dis)

call get_lun(nu_dis)
open(unit=nu_dis,file=f_dis(1:nc_f_dis),status='unknown')

write(nu_dis,'(5x,a, 3(4x,a, 4x,a, 1x,a, 4x,a, 1x,a))')
:           'time',
: 'dis_nf(1)', 'dis_if(1)', 'dis_nf_if(1)',
: 'dis_ff(1)', 'dis_total(1)',
: 'dis_nf(2)', 'dis_if(2)', 'dis_nf_if(2)',
: 'dis_ff(2)', 'dis_total(2)',
: 'dis_nf(3)', 'dis_if(3)', 'dis_nf_if(3)',
: 'dis_ff(3)', 'dis_total(3)'

nstart = int((tp-tp_minus_tstart)/dt) - 1
if (nstart .le. 0) then
  nstart = 1
end if
np = int(tp/dt)
ns = int(ts/dt)
npw = int(pw/dt)
nend = ns + npw + int(tend_minus_ts_plus_pw/dt) - 1

print *, ' tp, tp_minus_tstart, dt, nstart = ',
:      tp, tp_minus_tstart, dt, nstart
print *, ' ts, pw, tend_minus_ts_plus_pw, dt, nend = ',
:      ts, pw, tend_minus_ts_plus_pw, dt, nend

do i = nstart, np - 1

  time = float(i)*dt

  do j = 1, 3
    acc_nf(j) = 0.0
    acc_if(j) = 0.0
    acc_nf_if(j) = 0.0
    acc_ff(j) = 0.0
    acc_total(j) = 0.0
    vel_nf(j) = 0.0
    vel_if(j) = 0.0
    vel_nf_if(j) = 0.0
    vel_ff(j) = 0.0
    vel_total(j) = 0.0
    dis_nf(j) = 0.0
    dis_if(j) = 0.0
    dis_nf_if(j) = 0.0
    dis_ff(j) = 0.0

```



```

        dis_total(j) = 0.0
    end do

    write(nu_acc, '(1x,f8.3, 1p, 15(2x,e11.4))')
:       time, (acc_nf(j), acc_if(j), acc_nf_if(j), acc_ff(j),
:           acc_total(j), j = 1, 3)

    write(nu_vel, '(1x,f8.3, 1p, 15(2x,e11.4))')
:       time, (vel_nf(j), vel_if(j), vel_nf_if(j), vel_ff(j),
:           vel_total(j), j = 1, 3)

    write(nu_dis, '(1x,f8.3, 1p, 15(2x,e11.4))')
:       time, (dis_nf(j), dis_if(j), dis_nf_if(j), dis_ff(j),
:           dis_total(j), j = 1, 3)

end do

do i = np, nend

    time = float(i)*dt

    do j = 1, 3

        acc_nf(j) =
:           afctr_nf    * radpat_nf(j)    * anf(time, tp, ts, pw)

        acc_if(j) =
:           afctr_if_p * radpat_if_p(j) * aif(time, tp, pw)
:           + afctr_if_s * radpat_if_s(j) * aif(time, ts, pw)

        acc_nf_if(j) = acc_nf(j) + acc_if(j)

        acc_ff(j) =
:           afctr_ff_p * radpat_ff_p(j) * aff(time, tp, pw)
:           + afctr_ff_s * radpat_ff_s(j) * aff(time, ts, pw)

        acc_total(j) = acc_nf_if(j) + acc_ff(j)

        vel_nf(j) =
:           afctr_nf    * radpat_nf(j)    * vnf(time, tp, ts, pw)

        vel_if(j) =
:           afctr_if_p * radpat_if_p(j) * vif(time, tp, pw)
:           + afctr_if_s * radpat_if_s(j) * vif(time, ts, pw)

        vel_nf_if(j) = vel_nf(j) + vel_if(j)

        vel_ff(j) =
:           afctr_ff_p * radpat_ff_p(j) * vff(time, tp, pw)
:           + afctr_ff_s * radpat_ff_s(j) * vff(time, ts, pw)

        vel_total(j) = vel_nf_if(j) + vel_ff(j)

        dis_nf(j) =
:           afctr_nf    * radpat_nf(j)    * dnf(time, tp, ts, pw)

```

```

        dis_if(j) =
:         afctr_if_p * radpat_if_p(j) * dif(time, tp, pw)
:         + afctr_if_s * radpat_if_s(j) * dif(time, ts, pw)

        dis_nf_if(j) = dis_nf(j) + dis_if(j)

        dis_ff(j) =
:         afctr_ff_p * radpat_ff_p(j) * dff(time, tp, pw)
:         + afctr_ff_s * radpat_ff_s(j) * dff(time, ts, pw)

        dis_total(j) = dis_nf_if(j) + dis_ff(j)

    end do

    write(nu_acc, '(1x,f8.3, 1p, 15(2x,e11.4))')
:     time, (acc_nf(j), acc_if(j), acc_nf_if(j), acc_ff(j),
:           acc_total(j), j = 1, 3)

    write(nu_vel, '(1x,f8.3, 1p, 15(2x,e11.4))')
:     time, (vel_nf(j), vel_if(j), vel_nf_if(j), vel_ff(j),
:           vel_total(j), j = 1, 3)

    write(nu_dis, '(1x,f8.3, 1p, 15(2x,e11.4))')
:     time, (dis_nf(j), dis_if(j), dis_nf_if(j), dis_ff(j),
:           dis_total(j), j = 1, 3)

    end do

    close(nu_acc)
    close(nu_vel)
    close(nu_dis)
    close(nu_ctl)

    stop
    end
* -----
* -----
    function anf(t, tp, ts, pw)

* Near-field contribution to acceleration

* t = time from origin time
* r = hypocentral distance
* vs = S-wave velocity
* vp = P-wave velocity
* pw = duration of the pulse (pulse width)

    pp = t - tp
    ps = t - ts

    anf =          h(pp, pw) -          h(ps, pw)
:     + tp * hdot(pp, pw) - ts * hdot(ps, pw)

    return
    end

```

```

* -----
* -----
  function aif(t, tps, pw)
* Intermediate-field contribution to acceleration
* Do not need to know whether this is for the P- or the S-wave
* "tps" = tp or ts
  p = t - tps
  aif = hdotdot(p, pw)
  return
end
* -----
* -----
  function aff(t, tps, pw)
* Far-field contribution to acceleration
* Do not need to know whether this is for the P- or the S-wave
* "tps" = tp or ts
  p = t - tps
  aff = hdotdotdot(p, pw)
  return
end
* -----
* -----
  function vnf(t, tp, ts, pw)
* Near-field contribution to velocity
* t = time from origin time
* r = hypocentral distance
* vs = S-wave velocity
* vp = P-wave velocity
* pw = duration of the pulse (pulse width)
  pp = t - tp
  ps = t - ts
  vnf =      hint(pp, pw) -      hint(ps, pw)
:      + tp * h(pp, pw) - ts * h(ps, pw)
  return
end
* -----

```

```

* -----
  function vif(t, tps, pw)

* Intermediate-field contribution to velocity

* Do not need to know whether this is for the P- or the S-wave

* "tps" = tp or ts

  p = t - tps

  vif = hdot(p, pw)

  return
  end
* -----

* -----
  function vff(t, tps, pw)

* Far-field contribution to velocity

* Do not need to know whether this is for the P- or the S-wave

* "tps" = tp or ts

  p = t - tps

  vff = hdotdot(p, pw)

  return
  end
* -----

* -----
  function dnf(t, tp, ts, pw)

* Near-field contribution to displacement

* t = time from origin time
* r = hypocentral distance
* vs = S-wave velocity
* vp = P-wave velocity
* pw = duration of the pulse (pulse width)

  pp = t - tp
  ps = t - ts

  dnf =          hintint(pp, pw) - hintint(ps, pw)
:      + tp * hint(pp, pw) - ts * hint(ps, pw)

  return
  end
* -----

* -----
  function dif(t, tps, pw)

```

```

* Intermediate-field contribution to displacement
* Do not need to know whether this is for the P- or the S-wave
* "tps" = tp or ts
    p = t - tps
    dif = h(p, pw)
    return
end
* -----
* -----
    function dff(t, tps, pw)
* Far-field contribution to displacement
* Do not need to know whether this is for the P- or the S-wave
* "tps" = tp or ts
    p = t - tps
    dff = hdot(p, pw)
    return
end
* -----
* -----
    function h(p, pw)
    real h
    pi = 4.0*atan(1.0)
    twopi = 2.0*pi
    twopi_pw = twopi/pw
    if (p .lt. 0.0) then
        h = 0.0
    else if (p .ge. 0.0 .and. p .le. pw) then
        h = (p-sin(twopi_pw*p)/twopi_pw)/pw
    else
        h = 1.0
    end if
    return
end
* -----
* -----
    function hdot(p, pw)
    real hdot
    pi = 4.0*atan(1.0)

```

```

twopi = 2.0*pi
twopi_pw = twopi/pw

if (p .lt. 0.0 ) then
  hdot = 0.0
else if (p .ge. 0.0 .and. p .le. pw) then
  hdot = (1 - cos(twopi_pw*p))/pw
else
  hdot = 0.0
end if

return
end
* -----
* -----

function hdotdot(p, pw)
real hdot

pi = 4.0*atan(1.0)
twopi = 2.0*pi
twopi_pw = twopi/pw

if (p .lt. 0.0 ) then
  hdotdot = 0.0
else if (p .ge. 0.0 .and. p .le. pw) then
  hdotdot = (twopi_pw * sin(twopi_pw*p))/pw
else
  hdotdot = 0.0
end if

return
end
* -----
* -----

function hdotdotdot(p, pw)
real hdotdotdot

pi = 4.0*atan(1.0)
twopi = 2.0*pi
twopi_pw = twopi/pw

if (p .lt. 0.0 ) then
  hdotdotdot = 0.0
else if (p .ge. 0.0 .and. p .le. pw) then
  hdotdotdot = ((twopi_pw)**2 * cos(twopi_pw*p))/pw
else
  hdotdotdot = 0.0
end if

return
end
* -----
* -----

```

```

function hint(p, pw)
real hint

pi = 4.0*atan(1.0)
twopi = 2.0*pi
twopi_pw = twopi/pw

if (p .lt. 0.0 ) then
  hint = 0.0
else if (p .ge. 0.0 .and. p .le. pw) then
  hint = (p**2/2.0 - (1 - cos(twopi_pw*p))/twopi_pw**2)/pw
else
  hint = p - pw/2.0
end if

return
end
* -----
* -----
function hintint(p, pw)
real hintint

pi = 4.0*atan(1.0)
twopi = 2.0*pi
twopi_pw = twopi/pw

if (p .lt. 0.0 ) then
  hintint = 0.0
else if (p .ge. 0.0 .and. p .le. pw) then
  hintint = (- p/twopi_pw**2 + p**3/6.0
:           + (sin(twopi_pw*p))/twopi_pw**3)/pw
else
  hintint = (1.0/6.0 - 1.0/twopi**2) * pw**2 + p*(p - pw)/2.0
end if

return
end
* -----
* -----
function dot(a,b,n)

c Computes dot product between a and b

c Written by: Dave Boore
c Dates: 10/5/88 - Created

dimension a(1), b(1)

dot = 0.0

do 1 i = 1, n
1   dot = dot + a(i) * b(i)

```

```

    return
end
* -----
* -----
    subroutine unit_vector_fn(i_fn, sa, da, ra)
* Dates: 04/20/04 - Written by D. Boore

    real i_fn(3)

    pi = 4.0*atan(1.0)
    dtor = pi/180.0

    sar = sa*dtor
    dar = da*dtor
    rar = ra*dtor

    cos_sa = cos(sar)
    sin_sa = sin(sar)

    cos_da = cos(dar)
    sin_da = sin(dar)

    cos_ra = cos(rar)
    sin_ra = sin(rar)

    i_fn(1) = - sin_da*sin_sa
    i_fn(2) = + sin_da*cos_sa
    i_fn(3) = - cos_da

    return
end
* -----
* -----
    subroutine unit_vector_p(i_p, az, toa)
* Dates: 04/20/04 - Written by D. Boore

    real i_p(3)

    pi = 4.0*atan(1.0)
    dtor = pi/180.0

    azr = az*dtor
    toar = toa*dtor

    cos_az = cos(azr)
    sin_az = sin(azr)

    cos_toa = cos(toar)
    sin_toa = sin(toar)

    i_p(1) = + sin_toa*cos_az

```



```

    i_p(2) = + sin_toa*sin_az
    i_p(3) = + cos_toa

    return
end
* -----

* -----
    subroutine unit_vector_slip(i_slip, sa, da, ra)
* Dates: 04/20/04 - Written by D. Boore

    real i_slip(3)

    pi = 4.0*atan(1.0)
    dtor = pi/180.0

    sar = sa*dtor
    dar = da*dtor
    rar = ra*dtor

    cos_sa = cos(sar)
    sin_sa = sin(sar)

    cos_da = cos(dar)
    sin_da = sin(dar)

    cos_ra = cos(rar)
    sin_ra = sin(rar)

    i_slip(1) = cos_ra*cos_sa + cos_da*sin_ra*sin_sa
    i_slip(2) = cos_ra*sin_sa - cos_da*sin_ra*cos_sa
    i_slip(3) =                - sin_da*sin_ra

    return
end
* -----

include '\forprogs\Wholespace_Pointsource_util_subs.for'

```